Application of Derivatives

Question1

If the tangent of the curve $4y^3 = 3ax^2 + x^3$ drawn at the point (a, a) forms a triangle of area $\frac{25}{24}$ sq. units with the coordinates axes, then $a =$

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Options:

A.

± 10

B.

± 5

C.

± 6

D.

± 3

Answer: B

Solution:

$4y^3 = 3ax^2 + x^3 \qquad \ldots(i)$

A point (a, a) lies on the curve differentiating Eq. (i)

$12y^2 \cdot \frac{dy}{dx} = 6ax + 3x^2 \Rightarrow \frac{dy}{dx} = \frac{6ax+3x^2}{12y^2}$

at point (a, a)

$\frac{dy}{dx} = \frac{6a^2+3a^2}{12a^2} = 3/4$

So, equation of tangent

$(y-a)=\frac{3}{4}(x-a)$

For x-intercept, put $y=0$

$\Rightarrow \quad -a=\frac{3}{4}(x-a) \Rightarrow x=-a/3$

For y-intercept, put $x=0$

$\Rightarrow y-a=3/4(0-a) \Rightarrow y=a/4$

So, intercepts are $(-a/3,0)$ and $(0,a/4)$

So, area of the triangle

$=\frac{1}{2}\left|\frac{a}{3}\right| \cdot\left|\frac{a}{4}\right|=\frac{a^2}{24}$

and given that

$\frac{a^2}{24}=\frac{25}{24} \Rightarrow a=\pm 5$

Question2

If the function $f(x)=\sin x-\cos^2 x$ is defined on the interval $[-\pi,\pi]$, then f is strictly increasing in the interval

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Options:

A.

$\left(\frac{-5\pi}{6},\frac{-\pi}{6}\right)\cup\left(\frac{-\pi}{6},\frac{\pi}{2}\right)$

B.

$\left(\frac{-\pi}{2},\frac{-\pi}{6}\right)$

C.

$\left(\frac{-5\pi}{6},\frac{\pi}{2}\right)$

D.

$\left(\frac{-5\pi}{6},\frac{-\pi}{2}\right)\cup\left(\frac{-\pi}{6},\frac{\pi}{2}\right)$

Answer: D

Solution:

$f(x) = \sin x - \cos^2 x$
$f'(x) = \cos x(1 + 2\sin x)$

for $f(x)$ to be strictly increasing,

$f'(x) > 0 \Rightarrow \cos x(1 + 2\sin x) > 0$

if $\cos x > 0$ and $1 + 2\sin x > 0$

$x \in \left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$

and if $\cos x < 0$ and $1 + 2\sin x < 0$

$x \in (-5\pi/6, -\pi/2)$

Hence, $x \in \left(-\frac{5\pi}{6}, -\pi/2\right) \cup \left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$

Question3

If the Lagrange' mean value theorem is applied to the function $f(x) = e^x$ defined on the interval $[1, 2]$ and the value of $c \in (1, 2)$ is k, then $e^{k-1} =$

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Options:

A.

2

B.

$e - 1$

C.

$e + 1$

D.

Answer: B

Solution:

The Lagrange's Mean Value Theorem (LMVT) states that for a function $f(x)$ that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), there exists at least one point $c \in (a, b)$ such that:

$f'(c) = \frac{f(b)-f(a)}{b-a}$

Given the function $f(x) = e^x$ on the interval $[1, 2]$.

Here, $a = 1$ and $b = 2$.

First, let's check if the conditions for LMVT are met:

1. The function $f(x) = e^x$ is continuous on the closed interval $[1, 2]$ because the exponential function is continuous everywhere.
2. The function $f(x) = e^x$ is differentiable on the open interval $(1, 2)$ because its derivative $f'(x) = e^x$ exists everywhere.

Now, let's find the components for the LMVT formula:

1. Calculate the derivative $f'(x)$:

   $f'(x) = \frac{d}{dx}(e^x) = e^x$

2. Calculate $f(a)$ and $f(b)$:

   $f(a) = f(1) = e^1 = e$

   $f(b) = f(2) = e^2$

3. Apply the LMVT formula:

   $f'(c) = \frac{f(2)-f(1)}{2-1}$

   Substitute the values:

   $e^c = \frac{e^2-e}{1}$

   $e^c = e^2 - e$

The problem states that the value of $c \in (1, 2)$ is k. So, we can replace c with k:

$e^k = e^2 - e$

We need to find the value of e^{k-1}. We can rewrite e^{k-1} using exponent rules:

$e^{k-1} = \frac{e^k}{e^1} = \frac{e^k}{e}$

Now, substitute the expression for e^k into this equation:

$e^{k-1} = \frac{e^2-e}{e}$

Simplify the expression by dividing each term in the numerator by e:

$e^{k-1} = \frac{e^2}{e} - \frac{e}{e}$

$e^{k-1} = e^{2-1} - 1$

$e^{k-1} = e - 1$

Thus, the value of e^{k-1} is $e - 1$.

The final answer is $\boxed{e-1}$.

Question4

If the tangent of the curve $xy + ax + by = 0$ at $(1, 1)$ makes an angle $\tan^{-1} 2$ with X-axis, then $\frac{ab}{a+b} =$

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Options:

A.

1

B.

2

C.

3

D.

4

Answer: B

Solution:

Step 1: Put the given point into the equation.

The equation is $xy + ax + by = 0$. Plug in the point $(1, 1)$:

$1 \times 1 + a \times 1 + b \times 1 = 0$

$1+a+b=0$

So, $a+b=-1$ ... (i)

Step 2: Find the derivative to get the slope of the tangent.

We need to use implicit differentiation with respect to x:

$x\frac{dy}{dx}+y+a+b\frac{dy}{dx}=0$

Group the $\frac{dy}{dx}$ terms together:

$(x+b)\frac{dy}{dx}=-(a+y)$

So, $\frac{dy}{dx}=-\frac{a+y}{x+b}$

Step 3: Plug in the point $(1,1)$ to find the slope at that point.

At $(1,1)$: $\frac{dy}{dx}=-\frac{a+1}{b+1}$

We are told that the tangent makes an angle with the X-axis, and that angle is $\tan^{-1}2$. This means the slope at $(1,1)$ is 2.

So, $2=-\frac{a+1}{b+1}$

$2(b+1)=-(a+1)$

$2b+2=-a-1$

Rewrite: $a+2b=-3$ ... (ii)

Step 4: Solve the two equations to find a and b.

We have:
(i) $a+b=-1$
(ii) $a+2b=-3$

Subtract (i) from (ii):
$(a+2b)-(a+b)=(-3)-(-1)$
$a+2b-a-b=-3+1$
$b=-2$

Put $b=-2$ into (i):
$a+(-2)=-1$
$a=1$

Step 5: Find the required value $\frac{ab}{a+b}$.

$a=1, b=-2, a+b=-1.$

So, $\frac{ab}{a+b}=\frac{1\times(-2)}{-1}=\frac{-2}{-1}=2.$

Question5

If the displacement S of a particle travelling along a straight line in t seconds is given by $S = 2t^3 + 2t^2 - 2t - 3$, then the time taken (in second) by the particle to change its direction is

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Options:

A.

$\frac{1}{3}$

B.

2

C.

3

D.

$\frac{1}{2}$

Answer: A

Solution:

Step 1: Identify the displacement equation.

The displacement is given by $S = 2t^3 + 2t^2 - 2t - 3$.

Step 2: Find the velocity by differentiating displacement with respect to time.

The velocity is the derivative of displacement: $\frac{dS}{dt} = 6t^2 + 4t - 2$.

Step 3: Set the velocity equal to zero to find when the particle changes direction.

The particle changes direction when its velocity is zero: $6t^2 + 4t - 2 = 0$.

Step 4: Simplify and solve the quadratic equation.

Divide both sides by 2: $3t^2 + 2t - 1 = 0$. Factor the equation: $3t^2 + 3t - t - 1 = 0$. So, $3t(t + 1) - 1(t + 1) = 0$.

Step 5: Factor further and solve for t.

$(t + 1)(3t - 1) = 0$, so $t = -1$ or $t = \frac{1}{3}$.

Step 6: Choose the valid value for time.

Since time cannot be negative, the answer is $t = \frac{1}{3}$ seconds.

Question6

If the function $f(x) = x^3 + bx^2 + cx - 6$ satisfies all the conditions of Rolle's theorem in $[1, 3]$ and $f'\left(\frac{2\sqrt{3}+1}{\sqrt{3}}\right) = 0$, then $bc =$

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Options:

A.

18

B.

-66

C.

38

D.

-46

Answer: B

Solution:

Given, $f(x) = x^3 + bx^2 + cx - 6$

$\because$ Rolles theorem is applicable for $[1, 3]$

$\therefore f(1) = f(3)$
$\Rightarrow 1 + b + c - 6 = 27 + 9b + 3c - 6$
$\Rightarrow 8b + 2c = -26$
$\Rightarrow 4b + c = -13 \quad \ldots (i)$
$\therefore f'(x) = 3x^2 + 2bx + c$

$$f'\left(2 + \frac{1}{\sqrt{3}}\right) = 0$$

$$\Rightarrow 3\left(2 + \frac{1}{\sqrt{3}}\right)^2 + 2b\left(2 + \frac{1}{\sqrt{3}}\right) + c = 0$$

$$\Rightarrow 3\left(4 + \frac{1}{3} + \frac{4}{\sqrt{3}}\right) + 2b\left(2 + \frac{1}{\sqrt{3}}\right) + c = 0$$

$$\Rightarrow 2b\left(2 + \frac{1}{\sqrt{3}}\right) + c = -3\left(\frac{13}{3} + \frac{4}{\sqrt{3}}\right) \quad \ldots (ii)$$

Subtracting in Eqs. (i) and (ii), we get

$$2b\left(2 - 2 - \frac{1}{\sqrt{3}}\right) = -13 + 3\left(\frac{13}{3} + \frac{4}{\sqrt{3}}\right)$$

$\Rightarrow -2b/\sqrt{3} = 4\sqrt{3} \Rightarrow b = -6$
$c = -13 - 4b = -13 + 24 = 11$
$\therefore bc = -6 \times 11 = -66$

Question7

If the surface area of a spherical bubble is increasing at the rate of $4 \mathrm{sq.\,cm/sec}$, then the rate of change in its volume (in cubic $\mathrm{cm/sec}$) when its radius is 8 cms is

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Options:

A.

8

B.

12

C.

15

D.

16

Answer: D

Solution:

The surface area of sphere is $S=4\pi r^2, r$ is radius

Now, differentiate w.r.t x, we get

$\frac{ds}{dt}=4\pi\cdot 2r\cdot\frac{dr}{dt}=8\pi r\frac{dr}{dt}$

Also, given that $\frac{ds}{dt}=4\text{ cm}^2/\text{sec}$ and $r=8$ cm

$$\begin{aligned} &\therefore \quad \frac{ds}{dt}=\pi r\cdot\frac{dr}{dt}\Rightarrow 4=8\pi(8)\cdot\frac{dr}{dt} \\ &\Rightarrow \quad \frac{dr}{dt}=\frac{1}{16\pi}\text{ cm/sec} \qquad \ldots(i) \end{aligned}$$

Volume of Sphere is : $v=\frac{4}{3}\pi r^3$

Differentiate both sides w.r.t r, we get

$$\begin{aligned} \frac{dv}{dt} &= \frac{4}{3}\pi\cdot 3r^2\cdot\frac{dr}{dt}=4\pi\cdot(8)^2\cdot\frac{1}{16\pi}\text{ [using Eq. (i)]} \\ &= 16\text{ cm}^3/\text{sec} \end{aligned}$$

$\therefore$ Rate of change in the volume of the bubble $=16\text{ cm}^3/\text{sec}$

Question8

The number of turning points of the curve $f(x)=2\cos x-\sin 2x$ in the interval $[-\pi,\pi]$ is

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Options:

A.

4

B.

3

C.

1

D.

2

Answer: B

Solution:

Given curve, $f(x) = 2\cos x - \sin 2x$

$f'(x) = -2\sin x - 2\cos 2x$

To find turning points, set $f'(x) = 0$

$$\begin{aligned} &\Rightarrow \quad -2\sin x - 2\cos 2x = 0 \Rightarrow \sin x - \cos 2x = 0 \\ &\Rightarrow \quad \sin x + 1 - 2\sin^2 x = 0 \Rightarrow 2\sin^2 x - \sin x - 1 = 0 \end{aligned}$$

Let $u = \sin x$, then

$$\begin{aligned} & 2u^2 - u - 1 = 0 \Rightarrow (2u+1)(u-1) = 0 \\ \Rightarrow \quad & u = -\frac{1}{2} \text{ or } u = 1 \end{aligned}$$

$\therefore \sin x = \dfrac{-1}{2}$ or $\sin x = 1$

But $x \in [-\pi, \pi]$

So, $\sin x = \frac{-1}{2}$

$\Rightarrow \quad x = -\frac{5\pi}{6}, -\frac{\pi}{6}$

And $\sin x = 1 \Rightarrow x = \frac{\pi}{2}$

Thus, there are three solutions for $x \in [-\pi, \pi]$

$\therefore$ There are 3 turning points.

Question9

The radius and the height of a right circular solid cone are measured as 7 feet each. If there is an error of 0.002 ft for every feet in measuring them, then the error in the total surface area of the cone (in sq. ft) is

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Options:

A.

$(0.088)(\sqrt{2}+1)$

B.

$(0.616)(\sqrt{2}+1)$

C.

$(0.616)(\sqrt{2})$

D.

$(0.088)(\sqrt{2})$

Answer: B

Solution:

The total surface area of a cone is

$A=\pi r^2+\pi r l$, where r is radius and l is the slant height.

$$=\pi(7)^2+\pi(7)\cdot\sqrt{7^2+7^2} \quad \left[\because l=\sqrt{r^2+h^2} \text{ And } r=7, h=7\right]$$
$$=49\pi+49\sqrt{2}\pi=49\pi(1+\sqrt{2}) \quad \ldots(i)$$

But, the error in measuring r and h is $0.002\times 7=0.014$ feet

So, $dr=0.014$ and $dh=0.014$

Now, error in l, denoted by dl and differentiate it w.r.t r and h, we get

$dl=\frac{\partial l}{\partial r}dr+\frac{\partial l}{\partial h}\cdot dh \quad \ldots(ii)$

But $l=\sqrt{r^2+h^2}$

So, $\frac{\partial l}{\partial r}=\frac{1}{2}\left(r^2+h^2\right)^{-1/2}\cdot 2r=\frac{r}{\sqrt{r^2+h^2}}=\frac{r}{l}$

And $\frac{\partial l}{\partial h}=\frac{1}{2}\left(r^2+h^2\right)^{-1/2}\cdot 2h=\frac{h}{\sqrt{r^2+h^2}}=\frac{h}{l}$

So, $dl=\frac{r}{l}dr+\frac{h}{l}dh=\frac{7}{7\sqrt{2}}\cdot(0.014)+\frac{7}{7\sqrt{2}}(0.014)$

$= \frac{0.014}{\sqrt{2}} + \frac{0.014}{\sqrt{2}} = \frac{2}{\sqrt{2}}(0.014) = \sqrt{2}(0.014)$

The error in surface area, is dA, i.e.,

$dA = \frac{\partial A}{\partial r} dr + \frac{\partial A}{\partial l} dl$

Since, $A = \pi r^2 + \pi r l$

$$\therefore \quad \frac{\partial A}{\partial r} = 2\pi r + \pi l \text{ and } \frac{\partial A}{\partial l} = \pi r$$

$$\begin{aligned} dA &= (2\pi r + \pi l) \cdot dr + (\pi r) \cdot dl \\ &= (2\pi r + \pi l)(0 \cdot 0.14) + (\pi r)(0.014) \\ &= (14\pi + 7\sqrt{2}\pi)(0.014) + 7\sqrt{2}\pi(0.014) \\ &= 14\pi(0.014) + 7\sqrt{2}\pi(0.014) + 7\sqrt{2}\pi(0.014) \\ &= 0.196\pi + 0.098\sqrt{2}\pi + 0.098\sqrt{2}\pi \\ &= 0.196\pi + 0.196\sqrt{2}\pi \\ &= 0.196\pi(1 + \sqrt{2}) \\ &= (0.616)(\sqrt{2} + 1) \end{aligned}$$

Question10

The slope of a tangent drawn at the point $P(\alpha, \beta)$ lying on the curve $y = \frac{1}{2x-5}$ is -2 . If P lies in the fourth quadrant, then $\alpha - \beta =$

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Options:

A.

4

B.

3

C.

2

D.

1

Answer: B

Solution:

$y = \frac{1}{2x-5}$

So, $\frac{dy}{dx} = -\frac{1}{(2x-5)^2}(2) = \frac{-2}{(2x-5)^2}$ and given that

$\frac{-2}{(2\alpha-5)^2} = -2 \Rightarrow (2\alpha - 5)^2 = 1$

So, $2\alpha - 5 = \pm 1 \Rightarrow \alpha = 3, 2$

$\because P(\alpha, \beta)$ lie on the curve.

$\therefore \quad \beta = \frac{1}{2\alpha-5}$

at $\alpha = 2\beta = -1$

and when $\alpha = 3, \beta = 1$

$\because P$ lies in the fourth quadrant.

$\therefore$ Valid $R(\alpha, \beta) = (2, -1)$

Hence, $\alpha - \beta = 2 - (-1) = 3$

Question11

The function $f(x) = xe^{-x} \forall x \in R$ attains a maximum value at $x = k$, then $k =$

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Options:

A.

1

B.

2

C.

$\frac{1}{e}$

D.

3

Answer: A

Solution:

$\because f(x)=x\cdot e^{-x}$

$\therefore \quad f'(x)=e^{-x}-xe^{-x}=e^{-x}(1-x)$

for maximum value

$e^{-x}(1-x)=0\Rightarrow x=1\{\because e^{-x}\neq 0\}$

Now, $f''(x)=-e^{-x}+(1-x)(-e^{-x})$

at $x=1, f''(x)=-e^{-1}+0=-\frac{1}{e}<0$

Hence, $f(x)$ is maximum at $x=1$, therefore $k=1$.

Question12

If m and M are the absolute minimum and absolute maximum values of the function $f(x)=2\sqrt{2}\sin x-\tan x$ in the interval $[0,\pi/3]$, then $m+M=$

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Options:

A.

-1

B.

0

C.

1

D.

2

Answer: C

Solution:

$$f(x)=2\sqrt{2}\sin x-\tan x$$
$$\Rightarrow f'(x)=2\sqrt{2}\cos x-\sec^2 x$$

for critical point

$$2\sqrt{2}\cos x-\sin^2 x=0$$
$$\Rightarrow 2\sqrt{2}\cos x=\frac{1}{\cos^2 x}\Rightarrow \cos^3 x=\frac{1}{2\sqrt{2}}$$
$$\Rightarrow \quad \cos x=\frac{1}{\sqrt{2}}\Rightarrow x=\frac{\pi}{4}$$

Now, check $f(x)$ at $x=0,\frac{\pi}{4},\frac{\pi}{3}$

$f(0)=2\sqrt{2}\times 0-0=0$

$$f\left(\frac{\pi}{4}\right)=2\sqrt{2}\times\frac{1}{\sqrt{2}}-1=1$$
$$f\left(\frac{\pi}{3}\right)=2\sqrt{2}\times\frac{\sqrt{3}}{2}-\sqrt{3}=\sqrt{6}-\sqrt{3}<1$$

So, $m=0, M=1$

Hence, $m+M=0+1=1$

Question13

If $\frac{1}{2}\leq\frac{x^2+x+a}{x^2-x+a}\leq 2\forall x\in R$, then $a=$

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Options:

A.

$\frac{3}{4}$

B.

$\frac{-3}{4}$

C.

$\frac{9}{4}$

D.

$\frac{-9}{4}$

Answer: C

Solution:

Let $y = \frac{x^2+x+a}{x^2+2x+a} = \frac{x+\frac{a}{x}+1}{x+\frac{a}{x}+2}$

Let $g(x) = x + \frac{a}{x} \Rightarrow g'(x) = 1 - \frac{a}{x^2}$

For maximum or minimum put $g'(x) = 0$

$\Rightarrow \quad 1 - \frac{a}{x^2} = x \Rightarrow x = \pm\sqrt{a}$

Minimum value at $x = \sqrt{a}$ is $= \frac{2\sqrt{a}+1}{2\sqrt{a}+2}$

Maximum value at $x = -\sqrt{a}$ is

$$= \frac{2\sqrt{a}-1}{2\sqrt{a}-2}$$

Clearly, $\frac{2\sqrt{a}-1}{2\sqrt{a}-2} = 2$

$\Rightarrow \quad 2\sqrt{a} - 1 = 4\sqrt{a} - 4$

$\Rightarrow \quad 2\sqrt{a} = 3$

$\Rightarrow \quad \sqrt{a} = \frac{3}{2} \Rightarrow a = \frac{9}{4}$

Question14

P and Q are the ends of a diameter of the circle $x^2 + y^2 = a^2 \left(a > \frac{1}{\sqrt{2}}\right)$. s and t are the lengths of the perpendiculars drawn from P and Q onto the line $x + y = 1$ respectively. When the product st is maximum, the greater value among s, t is

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Options:

A.

$a+\sqrt{2}$

B.

$a+\frac{1}{\sqrt{2}}$

C.

$a-\frac{1}{\sqrt{2}}$

D.

$a-\sqrt{2}$

Answer: B

Solution:

Given, equation of circle is

$x^2+y^2=a^2 \qquad \ldots(i)$

Also P and Q are end points of diameter.

Let $P \equiv (a\cos\theta, a\sin\theta)$

$Q \equiv (-a\cos\theta, -a\sin\theta)$

According to the question,

$$\begin{aligned} s &= \frac{|a\cos\theta + a\sin\theta - 1|}{\sqrt{2}} \\ &= \frac{|a(\cos\theta+\sin\theta)-1|}{\sqrt{2}} \\ \text{and } t &= \frac{|-a(\cos\theta+\sin\theta)-1|}{\sqrt{2}} = 1 \\ \text{Now, st } &= \frac{|1-a^2(\cos\theta+\sin\theta)^2|}{2} \\ &= \frac{|1-a^2(1+\sin 2\theta)|}{2} \end{aligned}$$

So, (st) will be maximum, if $(1+\sin 2\theta)$ minimum,

$\because (\sin 2\theta)_{\min} = -1(\text{ i.e. } \sin 2\theta \geq -1)$
$\Rightarrow 1+\sin 2\theta \geq 0 \therefore (1+\sin 2\theta)_{\min} = 0$
$\therefore \left(a^2(1+\sin 2\theta)\right)_{\min} = 0$ and hence, $st = \frac{1}{2}$

Also, $\left(a^2(1+\sin 2\theta)\right)_{\max} = 2a^2$

$\Rightarrow \quad st = \frac{|1-2a^2|}{2} > \frac{1}{2}$ for $a > \frac{1}{\sqrt{2}}$

So, $(st)_{\max}$ occurs when

$$a^2(1+\sin 2\theta) = 2a^2$$
$$\Rightarrow \quad 1+\sin 2\theta = 2 \left(\because a > \frac{1}{\sqrt{2}} \therefore |a| = a\right)$$
$$\Rightarrow \quad \cos\theta + \sin\theta = \sqrt{2}$$
$$\Rightarrow \quad \theta = 45^\circ$$

$$\therefore \quad s = \frac{|\sqrt{2}a - 1|}{\sqrt{2}}, \text{ and } t = \frac{|\sqrt{2a+1}|}{\sqrt{2}}$$
$$\therefore \quad a > \frac{1}{\sqrt{2}}$$
$$\Rightarrow \quad \sqrt{2}a > 1$$
$$\therefore \quad s = \frac{\sqrt{2}a - 1}{\sqrt{2}},$$
$$t = \frac{\sqrt{2}a+1}{\sqrt{2}} = a + \frac{1}{\sqrt{2}}$$

Clearly, $t > s$

$\therefore \quad \left(a + \frac{1}{\sqrt{2}}\right)$ is the final answer.

Question15

Let $P(x) = x^4 + ax^3 + bx^2 + cx + d$ be such that $x = 0$ is the only real root of $P^1(x) = 0$. If $P(-1)
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Options:

A.

$P(-1)$ is not minimum of $P(x)$, but $P(1)$ is the maximum of $P(x)$

B.

$P(-1)$ is minimum of $P(x)$, but $P(1)$ is not the maximum of $P(x)$

C.

Neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of $P(x)$

D.

$P(-1)$ is the minimum and $P(1)$ is the maximum of $P(x)$

Answer: A

Solution:

Given,

$$P(x)=x^4+ax^3+bx^2+cx+d$$
$$\Rightarrow \quad P'(x)=4x^3+3ax^2+2bx+c$$

$\because x=0$ is the only real root of $P'(x)$ means the cubic equation.

$4x^3+3ax^2+2bx+c=0$

$\therefore x=0$ must be a triple root, i.e. all other roots are complex or multiplicity is 3 .

Hence, $P'(x)=4x^3$ i.e. $3a=0 \Rightarrow a=0$, $2b=0 \Rightarrow b=0$ and $c=0$

So, $P(x)=x^4+d$

Now, $P(-1)=(-1)^4+d=1+d$

$P(1)=(1)^4+d=1+d$

Thus, $P(-1)=P(1)$ which contradicting the given condition $P(-1)

So, our assumption that 0 is a triple root must be false.

So, $P'(x)=0$ has exactly one real root and two complex conjugate roots.

It means the sign of $P'(x)=0$ does not change around $x=0$ (because there is only one real root)

Hence, $P'(x)<0$ on $(-1,0)$ and $P'(x)>0$ on $(0,1)$

i.e. on the interval $[-1,1]$, the function has a minimum at $x0$.

and is strictly decreasing from (-1) to 0 and then strictly increasing from 0 to 1 .

So, option (a) will be true.

Question16

If the volume of a sphere is increasing at the rate of 12 c.c. $/\mathrm{sec}$, then the rate (in $\mathrm{sq.\ cm/sec}$) at which its surface area is increasing, when the diameter of the sphere is 12 cm is

Options:

A.

2

B.

3

C.

4

D.

6

Answer: C

Solution:

As we know that, volume of sphere

$= v = \frac{4}{3}\pi r^3$

and Surface area of sphere $= s = 4\pi r^2$

Now, $\frac{d\nu}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\Rightarrow \quad \frac{dr}{dt} = \frac{1}{4\pi r^2} \times \frac{dv}{dt}$

Putting $r = 6$ and $\frac{dv}{dt} = 12$

($\because$ Diameter of sphere $= 12$ cm (given)

$\therefore$ Radius of sphere $= 6$ cm)

$\frac{dr}{dt} = \frac{12}{4\pi(6)^2} = \frac{1}{12\pi}$

Now, $\frac{ds}{dt} = 8\pi r \cdot \frac{dr}{dt}$

$\therefore \quad \frac{ds}{dt} = 8\pi \times 6 \times \frac{1}{12\pi} = 4 \text{ cm}^2$

(putting $r = 6$ and $\frac{dr}{dt} = \frac{1}{12\pi}$)

Question17

If the lengths of the tangent, subtangent, normal and subnormal for the curve $y = x^2 + x - 1$ at the point $(1, 1)$ are a, b, c and d respectively, then their increasing order is

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Options:

A.

b, d, a, c

B.

b, a, c, d

C.

a, b, c, d

D.

b, a, d, c

Answer: D

Solution:

Given equation of curve is

$$y = x^2 + x - 1 \qquad \ldots (i)$$

$$\Rightarrow \frac{dy}{dx} = 2x + 1$$

$$\therefore \left.\frac{dy}{dx}\right|_{x=1} = 2 \times 1 + 1 = 3 = m$$

i.e. slope of tangent $= m = 3$. and point is $(1, 1)$ (by putting $x = 1$ in Eq. (i))

Now, length of tangent

$$= a = \left|\frac{y\sqrt{1 + m^2}}{m}\right|$$

$$= \left|\frac{1\sqrt{1 + 3^2}}{3}\right| = \frac{\sqrt{10}}{3}$$

length of subtangent

$= b = \left|\frac{y}{m}\right| = \left|\frac{1}{3}\right| = \frac{1}{3}$

length of normal

$= c = \left|y\sqrt{1+m^2}\right| = |1 \times \sqrt{10}| = \sqrt{10}$

length of subnormal

$= d = |y \cdot m| = |3| = 3$

Now, from above result, we conclude that $b

Question18

If the tangent drawn at the point $(x_1, y_1), x_1, y_1 \in N$ on the curve $y = x^4 - 2x^3 + x^2 + 5x$ passes through origin, then $x_1 + y_1 =$

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Options:

A.

5

B.

4

C.

7

D.

6

Answer: D

Solution:

$$y = x^4 - 2x^3 + x^2 + 5x$$
$$\Rightarrow \frac{dy}{dx} = 4x^3 - 6x^2 + 2x + 5$$

The slope of tangent line at (x_1, y_1) is

$$m = 4x_1^3 - 6x_1^2 + 2x_1 + 5$$

The equation of tangent line is

$$y - y_1 = \left(4x_1^3 - 6x_1^2 + 2x_1 + 5\right)(x - x_1)$$

Since, the tangent line passes through $(0, 0)$

$$\Rightarrow 0 - y_1 = \left(4x_1^3 - 6x_1^2 + 2x_1 + 5\right)(0 - x_1)$$
$$y_1 = \left(4x_1^3 - 6x_1^2 + 2x_1 + 5\right)x_1$$

Since, (x_1, y_1) is on the curve

$$y_1 = x_1^4 - 2x_1^3 + x_1^2 + 5x_1$$

Substituting this into the previous equation, we get

$$x_1^4 - 2x_1^3 + x_1^2 + 5x_1$$
$$= \left(4x_1^3 - 6x_1^2 + 2x_1 + 5\right)x_1$$
$$\Rightarrow x_1^4 - 2x_1^3 + x_1^2 + 5x_1$$
$$= 4x_1^4 - 6x_1^3 + 2x_1^2 + 5x_1$$
$$\Rightarrow 3x_1^4 - 4x_1^3 + x_1^2 = 0$$
$$\Rightarrow x_1^2\left(3x_1^2 - 4x_1 + 1\right) = 0$$
$$\Rightarrow x_1^2(3x_1 - 1)(x_1 - 1) = 0$$
$$\Rightarrow x_1 = 0, x_1 = \frac{1}{3}, x_1 = 1$$
$$x_1 = 1, y_1 = 5 \Rightarrow x_1 + y_1 = 1 + 5 = 6$$

Question19

Which one of the following functions is monotonically increasing in its domain?

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Options:

A.

$f(x) = \log(1 + x) - x + \frac{x^2}{2}$

B.

$g(x) = 2\tan^{-1}x - x - 1$

C.

$h(x) = 4\cos x + x$

D.

$u(x) = \log(1 + x) - \frac{x}{x+1}$

Answer: A

Solution:

From the given, we have

$f(x) = \log(1 + x) - x + \frac{x^2}{2}$

Differentiating both sides, we get $\Rightarrow f'(x) = \frac{1}{(1+x)} - 1 + x$

$$\Rightarrow f'(x) = \left\{\frac{1 - 1 - x + x + x^2}{(1 + x)}\right\}$$
$$\Rightarrow f'(x) = \frac{x^2}{(1 + x)}$$

Hence, $f(x)$ is monotonically increasing in its domain.

Question20

If β is an angle between the normals drawn to the curve $x^2 + 3y^2 = 9$ at the points $(3\cos\theta, \sqrt{3}\sin\theta)$ and $(-3\sin\theta, \sqrt{3}\cos\theta), \theta \in \left(0, \frac{\pi}{2}\right)$, then

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Options:

A.

$\tan\beta = \frac{1}{\sqrt{3}}\sec 2\theta$

B.

$\cot \beta = \sqrt{3} \operatorname{cosec} 2\theta$

C.

$\sqrt{3} \cot \beta = \sin 2\theta$

D.

$\cot \beta = \frac{1}{\sqrt{2}} \sec 2\theta$

Answer: C

Solution:

Step 1: Find the Derivative and Slope of Normal

The curve is $x^2 + 3y^2 = 9$.

Differentiate both sides with respect to x:

$2x + 3 \cdot 2y \cdot y' = 0$

Solve for y':

$2x + 6yy' = 0 \implies 6yy' = -2x \implies y' = -\frac{2x}{6y} = -\frac{x}{3y}$

The slope of the tangent is $y' = -\frac{x}{3y}$, so the slope of the normal is $-\frac{1}{y'} = \frac{3y}{x}$.

Step 2: Find the Slope of Normals at Given Points

First point: $(3 \cos \theta, \sqrt{3} \sin \theta)$

Slope at this point:

$m_1 = \frac{3 \cdot \sqrt{3} \sin \theta}{3 \cos \theta} = \sqrt{3} \tan \theta$

Second point: $(-3 \sin \theta, \sqrt{3} \cos \theta)$

Slope at this point:

$m_2 = \frac{3 \cdot \sqrt{3} \cos \theta}{-3 \sin \theta} = -\sqrt{3} \cot \theta$

Step 3: Find the Angle Between Normals (β)

Use the formula for the tangent of the angle between two lines:

$\tan \beta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$\tan \beta = \left| \frac{\sqrt{3} \tan \theta - (-\sqrt{3} \cot \theta)}{1 + (\sqrt{3} \tan \theta)(-\sqrt{3} \cot \theta)} \right|$

$\tan\beta = \left|\frac{\sqrt{3}(\tan\theta+\cot\theta)}{1-3}\right|$

$\tan\beta = \left|\frac{\sqrt{3}(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta})}{-2}\right|$

$\tan\beta = \frac{\sqrt{3}}{2}\cdot\frac{1}{\sin\theta\cos\theta}$

Recall $2\sin\theta\cos\theta = \sin 2\theta$, so:

$\tan\beta = \frac{\sqrt{3}}{\sin 2\theta}$

This means $\frac{1}{\cot\beta} = \frac{\sqrt{3}}{\sin 2\theta}$ or $\sqrt{3}\cot\beta = \sin 2\theta$.

Question21

If the area of a right-angle triangle with hypotenuse 5 is maximum, then its perimeter is

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Options:

A.

12

B.

$2\sqrt{3}+\sqrt{13}+5$

C.

$7+\sqrt{21}$

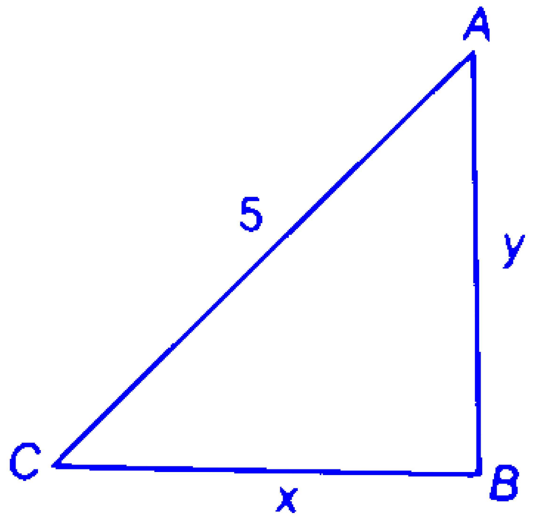
D.

$5(\sqrt{2}+1)$

Answer: D

Solution:

$x^2 + y^2 = 5^2$
$y^2 = 5^2 - x^2 \Rightarrow y = \sqrt{5^2 - x^2}$
$0 \leq x \leq 5$



$$\text{Area } = \frac{1}{2} \text{ base } \times \text{ height}$$
$$A = \frac{1}{2}xy = \frac{1}{2}x\sqrt{25 - x^2}$$
$$\frac{dA}{dx} = \frac{1}{2}\left(25 - x^2\right)^{-\frac{1}{2}}\left(25 - 2x^2\right)$$
$$= \frac{25 - 2x^2}{2\sqrt{25 - x^2}}$$
$$\Rightarrow \quad \frac{dA}{dx} = 0 \Rightarrow \frac{25 - 2x^2}{2\sqrt{25 - x^2}} = 0$$
$$\Rightarrow \quad 25 - 2x^2 = 0$$
$$\Rightarrow \quad 25 = 2x^2$$
$$\Rightarrow \quad x = \frac{5}{\sqrt{2}}, y = \frac{5}{\sqrt{2}}$$
$$\Rightarrow \quad AB = \frac{5}{\sqrt{2}}, BC = \frac{5}{\sqrt{2}}, AC = 5$$
$$\text{Perimeter } = AB + BC + AC$$
$$= \frac{5}{\sqrt{2}} + \frac{5}{\sqrt{2}} + 5 = 5\sqrt{2} + 5$$
$$= 5(\sqrt{2} + 1)$$

Question22

If $y = |\cos x - \sin x| + |\tan x - \cot x|$, then

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} + \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{6}} =$$

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Options:

A.

1

B.

-1

C.

2

D.

0

Answer: D

Solution:

$y = |\cos x - \sin x| + |\tan x - \cot x|$

For $''x = \frac{\pi}{3}$

$$\begin{aligned} y &= -(\cos x - \sin x) + (\tan x - \cot x) \\ &= \cos x + \sin x + \tan x - \cot x \end{aligned}$$

For $x = \frac{\pi}{6}$

$$\begin{aligned} y &= (\cos x - \sin x) - (\tan x - \cot x) \\ &= \cos x - \sin x - \tan x + \cot x \end{aligned}$$

$\frac{dy}{dx}]_x = \frac{\pi}{3}$

$= \sin\frac{\pi}{3} + \cos\frac{\pi}{3} + \sec^2\frac{\pi}{3} + \operatorname{cosec}^2\frac{\pi}{3}$

$= \frac{35 + 3\sqrt{3}}{6}$

$$\left.\frac{dy}{dx}\right]_{x=\frac{\pi}{6}}$$
$$= -\sin\frac{\pi}{6} - \cos\frac{\pi}{6} - \sec^2\frac{\pi}{6} - \operatorname{cosec}^2\frac{\pi}{6}$$
$$= \frac{-35 - 3\sqrt{3}}{6}$$
$$\left.\frac{dy}{dx}\right]_{x=\frac{\pi}{3}} + \left.\frac{dy}{dx}\right]_{x=\frac{\pi}{6}} = 0$$

Question23

If the tangent drawn at the point (α, β) on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ is parallel to the line $\sqrt{3x} + y = 1$, then $\alpha^2 + \beta^2 =$

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Options:

A.

10

B.

9

C.

28

D.

19

Answer: C

Solution:

Given curve, $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4 \quad \ldots(i)$

and line $y = -\sqrt{3}x + 1 \quad \ldots(ii)$

From Eq. (i), $\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}y' = 0$

$$y'l_{(\alpha,\beta)} = -\left(\frac{\beta}{\alpha}\right)^{\frac{1}{3}}$$

$$-\left(\frac{\beta}{\alpha}\right)^{\frac{1}{3}} = -\sqrt{3} \Rightarrow \frac{\beta}{\alpha} = 3\sqrt{3}$$

$$\beta = 3\sqrt{3}\alpha$$

From Eq. (i) $\alpha^{\frac{2}{3}} + 3\alpha^{\frac{2}{3}} = 4 \Rightarrow \alpha = \pm 1$

$$\beta = \pm 3\sqrt{3}$$

$$\therefore \quad \alpha^2 + \beta^2 = 28$$

Question24

The displacement S of a particle measured from a fixed point O on a line is given by $S = t^3 - 16t^2 + 64t - 16$. Then, the time at which displacement of the particle is maximum is

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Options:

A.

8

B.

4

C.

$\frac{8}{3}$

D.

$\frac{4}{3}$

Answer: C

Solution:

$S = t^3 - 16t^2 + 64t - 6$

$\Rightarrow \frac{dS}{dt} = 3t^2 - 32t + 64$

For maxima/minima $\frac{dS}{dt} = 0$

$$\Rightarrow \quad (3t^2 - 8)(t - 8) = 0 \Rightarrow t = 8, \frac{8}{3}$$
$$\frac{d^2s}{dt^2} = 6t - 32 \Rightarrow \left.\frac{d^2s}{dt^2}\right]_{t=\frac{8}{3}} < 0$$

So, required time, $t = \frac{8}{3}$

Question25

If the extreme value of the function $f(x) = \frac{4}{\sin x} + \frac{1}{1-\sin x}$ in $\left[0, \frac{\pi}{2}\right]$ is m and it exists at $x = k$, then $\cos k =$

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Options:

A.

$\frac{\sqrt{m}}{4}$

B.

$\frac{\sqrt{m+1}}{\sqrt{2}}$

C.

$\frac{\sqrt{5}}{\sqrt{m}}$

D.

$\frac{1}{m}$

Answer: C

Solution:

$$f(x)=\frac{4}{\sin x}+\frac{1}{1-\sin x}$$
$$\Rightarrow f'(x)=\frac{-4\cos x}{\sin^2 x}+\frac{\cos x}{(1-\sin x)^2}$$
$$f'(x)=0$$
$$\Rightarrow \cos x\left[\frac{1}{(1-\sin x)^2}-\frac{4}{\sin^2 x}\right]=0$$
$$\Rightarrow \cos x\left[\frac{\sin^2 x-4(1-\sin x)^2}{(\sin^2 x)(1-\sin x)^2}\right]=0$$
$$\Rightarrow \cos x[(\sin x+2-2\sin x)(\sin x-2+2\sin x)]$$
$$=0$$
$$\Rightarrow \quad x=\frac{\pi}{2}, \sin x=2 \sin x=\frac{2}{3}$$
$$f(0)=\text{ Not possible}$$
$$f\left(\frac{\pi}{2}\right)=\text{ Not possible}$$
$$\sin x\neq 2 \quad (\because \sin x\in[-1,1])$$
$$\therefore \quad \sin x=\frac{2}{3}$$
$$f(x)l_{\max}=6+3=9=m$$
$$\Rightarrow \quad \cos k=\sqrt{1-\frac{4}{9}}=\sqrt{\frac{5}{9}}=\sqrt{\frac{5}{m}}$$

Question26

If the normal drawn at the point P on the curve $y=x\log x$ is parallel to the line $2x-2y=3$, then $P=$

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Options:

A.

(e, e)

B.

$\left(\frac{1}{e}, \frac{-1}{e}\right)$

C.

$\left(\frac{1}{e^2}, \frac{-2}{e^2}\right)$

D.

$\left(e^3, 3e^3\right)$

Answer: C

Solution:

Let $P(h, k)$

$$\begin{aligned} & y = x \log x \\ \Rightarrow \quad & \frac{dy}{dx} = 1 + \log x \\ & \therefore \text{ Slope of normal } = -\frac{1}{1+\log h} \end{aligned}$$

And slope of the given line $2x - 2y = 3$ is 1

$$\begin{aligned} & \Rightarrow \quad -\frac{1}{1+\log h} = 1 \\ & \Rightarrow \quad 1 + \ln h = -1 \\ & \Rightarrow \ln h = -2 \end{aligned}$$

$h = e^{-2}$

$k = h \ln h$

$k = e^{-2} \ln e^{-2} = e^{-2}(-2) = -\frac{2}{e^2}$

$\therefore$ The point P is $\left(\frac{1}{e^2}, -\frac{2}{e^2}\right)$

Question27

If the curves $y^2 = 16x$ and $9x^2 + \alpha y^2 = 25$ intersect at right angles, then $\alpha =$

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Options:

A.

6

B.

9

C.

$\frac{9}{2}$

D.

3

Answer: C

Solution:

We have the curve

$y^2 = 16x$ and $9x^2 + \alpha y^2 = 25$

On differentiating, we get slopes

$2yy' = 16$ and $18x + 2\alpha yy' = 0$

$y' = \frac{8}{y}, y' = -\frac{9x}{\alpha y}$

Clearly, $m_1 = \frac{8}{y}$ and $m_2 = -\frac{9x}{\alpha y}$

$$m_1 m_2 = -1$$
$$\frac{8}{y} \times \left(\frac{-9x}{\alpha y}\right) = -1 \Rightarrow -\frac{72x}{\alpha y^2} = -1$$
$$72x = \alpha y^2 = \alpha \cdot 16x$$
$$16\alpha = 72$$
$$\alpha = \frac{72}{16} = \frac{9}{2}$$
$$\therefore \quad \alpha = \frac{9}{2}$$

Question28

If the function $y = \sin x(1 + \cos x)$ is defined in the interval $[-\pi, \pi]$, then y is strictly increasing in the interval

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Options:

A.

$\left(-\pi,-\frac{\pi}{3}\right) \cup\left(\frac{\pi}{3}, \pi\right)$

B.

$\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$

C.

$\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

D.

$\left(-\pi,-\frac{\pi}{6}\right) \cup\left(\frac{\pi}{6}, \pi\right)$

Answer: C

Solution:

We have, $y=\sin x(1+\cos x)$

$$\begin{aligned} \frac{d y}{d x} & =\sin x(-\sin x)+(1+\cos x) \cos x \\ & =-\sin ^2 x+\cos x+\cos ^2 x \\ & =-1+\cos ^2 x+\cos x+\cos ^2 x \\ & =2 \cos ^2 x+\cos x-1 \end{aligned}$$

$$\Rightarrow \frac{d y}{d x}=(\cos x+1)(2 \cos x-1)$$

Given y is strictly increasing

$$\therefore \quad \frac{d y}{d x}=(\cos x+1)(2 \cos x-1)>0$$

$$\Rightarrow \cos x<-1 \text { or } \cos x>\frac{1}{2}$$

$$\Rightarrow x \in\left(-\frac{\pi}{3}, \frac{\pi}{3}\right) \text { in the interval }(-\pi, \pi)$$

$$\therefore x \in\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$$

Question29

If the velocity of a particle moving on a straight line is proportional to the cube root of its displacement, then its acceleration is

Options:

A.

constant

B.

inversely proportional to its velocity

C.

proportional to its velocity

D.

proportional to its displacement

Answer: B

Solution:

Given:

Velocity v is **proportional to the cube root of the displacement** x.

$v \propto x^{1/3}$

This means we can write:

$v = kx^{1/3}$

where k is a constant of proportionality.

We are asked to find how acceleration a depends on other quantities.

We know that:

$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v\frac{dv}{dx}$

Step 1: Find $\frac{dv}{dx}$

From $v = kx^{1/3}$,

$\frac{dv}{dx} = k \cdot \frac{1}{3}x^{-2/3} = \frac{k}{3}x^{-2/3}$

Step 2: Substitute in acceleration formula:

$a = v\frac{dv}{dx} = (kx^{1/3}) \cdot \left(\frac{k}{3}x^{-2/3}\right)$

Simplify:

$a = \frac{k^2}{3}x^{1/3-2/3} = \frac{k^2}{3}x^{-1/3}$

Step 3: Express in terms of velocity if desired

Since $v = kx^{1/3} \Rightarrow x^{1/3} = \frac{v}{k}$,

$x^{-1/3} = \frac{k}{v}$

So:

$a = \frac{k^2}{3} \cdot \frac{k}{v} = \frac{k^3}{3v}$

Therefore:

$a \propto \frac{1}{v}$

✅ **Final Answer:**

Option B: *inversely proportional to its velocity*

Question30

If α and $\beta(\alpha > \beta)$ are the multiple roots of the equation $4x^4 + 4x^3 - 23x^2 - 12x + 36 = 0$, then $2\alpha - \beta =$

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Options:

A.

-1

B.

3

C.

5

D.

-7

Answer: C

Solution:

We have, $f(x)=4x^4+4x^3-23x^2-12x+36=0$

Using concept a polynomial $f(x)$ has a multiple root r, then $f(r)=0$ and

$f'(r)=0$
$f'(x)=16x^3+12x^2-46x-12$

Now, using hit and trial

By taking $x=-2$

$$
\begin{aligned}
f(-2) &= 4(-2)^4+4(-2)^3-23(-2)^2 \\
&\quad -12(-2)+36 \\
&= 64-32-92+24+36=0 \\
\Rightarrow f'(-2) &= 16(-2)^3+12(-2)^2 \\
&\quad -46(-2)-12 \\
&= -128+48+92-12=0
\end{aligned}
$$

$\because \quad x=-2$ is a multiple root.

By dividing the equation by $(x+2)^2$, we get another multiple root $x=\frac{3}{2}$

Given, $\alpha>\beta$

$\therefore \quad \alpha=\dfrac{3}{2}$ and $\beta=-2$

$\therefore 2\alpha-\beta=3+2=5$

Question31

The area (in square units) of the triangle formed by the X-axis, the tangent and the normal drawn at $(1,1)$ to the curve $x^3+y^3=2xy$ is

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Options:

A.

$1/2$

B.

1

C.

2

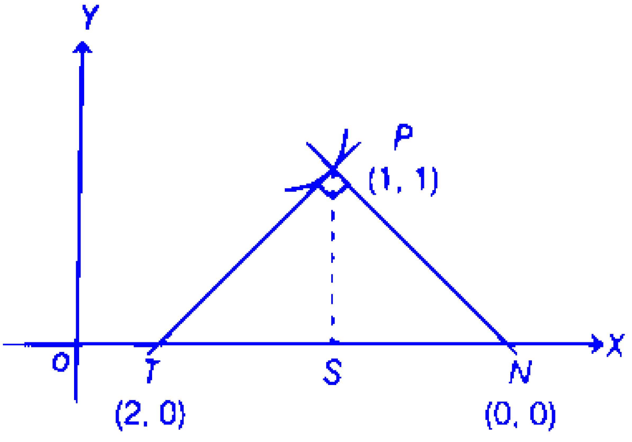
D.

3/2

Answer: B

Solution:

We have, the curve
$x^3 + y^3 = 2xy$



$$\Rightarrow \quad 3x^2 + 3y^2y' = 2xy' + y \cdot 2$$
$$\Rightarrow \quad y'\left(3y^2 - 2x\right) = 2y - 3x^2$$
$$\Rightarrow \quad y' = \frac{2y - 3x^2}{3y^2 - 2x}$$

Slope of tangent $= \left(\frac{dy}{dx}\right)_{(1,1)} = \frac{2-3}{3-2}$

$= \frac{-1}{1} = -1$

Slope of normal $= 1$

Equation of tangent $\Rightarrow y - 1 = -1(x - 1)$

$y - 1 = -x + 1$
$x + y = 2$

x-intercept of tangent $= (2, 0)$

Equation of normal,

$y-1=1(x-1)$
$y=x$

$\because$ Area of $\triangle PTN=\frac{1}{2}\times NT\times PS$

$=\frac{1}{2}\times 2\times 1=1$ sq. unit

Question32

The value of the Rolle's theorem for the function $f(x)=2\sin x+\sin 2x$ in the interval $[0,\pi]$ is

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Options:

A.

$\frac{\pi}{2}$

B.

$\frac{\pi}{6}$

C.

$\frac{\pi}{4}$

D.

$\frac{\pi}{3}$

Answer: D

Solution:

$f(x)=2\sin x+\sin 2x$

$f(x)$ is continuous and differentiable

$f(0)=f(\pi)$

$\therefore$ Rolle's Theorem is applicable

$$f'(x) = 2\cos x + \cos 2x \cdot 2$$
$$\Rightarrow f'(c) = 2\cos c + 2\left(2\cos^2 c - 1\right) = 0$$
$$= 2\left[2\cos^2 c + \cos c - 1\right] = 0$$
$$= 2\cos^2 c + 2\cos c - \cos c - 1 = 0$$
$$\Rightarrow 2\cos c(\cos c + 1) - 1(\cos c + 1) = 0$$
$$\Rightarrow (\cos c + 1)(2\cos c - 1) = 0$$
$$\Rightarrow \cos c = -1 \text{ and } \cos c = \frac{1}{2}$$
$$\because \quad c = \frac{\pi}{3}$$

Question33

If the function $y = g(x)$ representing the slopes of the tangents drawn to the curve $y = 3x^4 - 5x^3 - 12x^2 + 18x + 3$ is strictly increasing, then the domain of $g(x)$ is

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Options:

A.

$\left[-\frac{1}{2}, \frac{4}{3}\right]$

B.

$\left(\frac{-1}{2}, \frac{4}{3}\right)$

C.

$R - \left(\frac{-1}{2}, \frac{3}{4}\right)$

D.

$R - \left[\frac{-1}{2}, \frac{4}{3}\right]$

Answer: D

Solution:

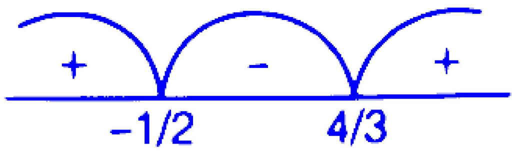
$y = 3x^4 - 5x^3 - 12x^2 + 18x + 3$

Slope of tangent

$$g(x)=\frac{dy}{dx}=12x^3-15x^2-24x+18$$
$$g'(x)=36x^2-30x-24$$
$$=6\left(6x^2-5x-4\right)$$

$\therefore g(x)$ is increasing $\quad [\because g'(x)>0]$

$$6x^2-5x-4>0$$
$$6x^2-8x+3x-4>0$$
$$2x(3x-4)+1(3x-4)>0$$
$$(3x-4)(2x+1)>0$$



$$x<-\frac{1}{2} \text{ or } x>\frac{4}{3}$$
$$\therefore \quad R-\left[-\frac{1}{2},\frac{4}{3}\right]$$

Question34

A is a point on the circle with radius 8 and centre at O. A particle P is moving on the circumference of the circle starting from A. M is the foot of the perpendicular from P on OA and $\angle POM=\theta$. When $OM=4$ and $\frac{d\theta}{dt}=6$ radians /sec, then the rate of change of PM is (in units/sec)

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Options:

A. $24\sqrt{3}$

B. 24

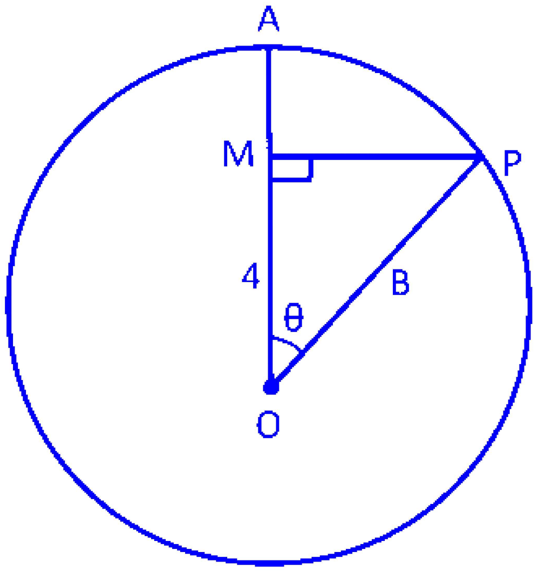
C. $15\sqrt{3}$

D. $48\sqrt{3}$

Answer: B

Solution:

In $\triangle POM$,



$$\cos\theta = \frac{OM}{OP} = \frac{4}{8} = \frac{1}{2}$$
$$\sin\theta = \frac{PM}{OP} = \frac{PM}{8}$$

On differentiating both sides w.r.t.t, we get

$$\cos\theta \frac{d\theta}{dt} = \frac{1}{8}\frac{d(PM)}{dt}$$
$$\frac{4}{8} \times 6 = \frac{1}{8} \times \frac{d(PM)}{dt}$$
$$\Rightarrow \quad \frac{d(PM)}{dt} = 24$$

Question35

If the length of the sub-tangent at any P on a curve is proportional to the abscissa of the point P, then the equation of that curve is (C

is an arbitrary constant)

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Options:

A. $y^k + x^k = C$

B. $x^{\frac{1}{k}} C = y^k$

C. $(x + y)^k = C$

D. $y = x^{\frac{1}{k}} C$

Answer: D

Solution:

Given that the length of the sub-tangent at any point P on a curve is proportional to the abscissa x of the point P, we start by expressing this relationship mathematically:

$\frac{y}{\frac{dy}{dx}} \propto x$

This implies:

$\frac{y}{\frac{dy}{dx}} = kx$

where k is the constant of proportionality. By rearranging, we get:

$\frac{dy}{y} = \frac{dx}{kx}$

Integrating both sides, we obtain:

$\int \frac{dy}{y} = \int \frac{dx}{kx}$

which results in:

$\log y = \frac{1}{k} \log x + \log c$

Rearranging this:

$\log y = \log x^{1/k} + \log c$

Therefore, we have:

$\log y = \log(x^{1/k} c)$

Exponentiating both sides gives us:

$y = x^{1/k} c$

Thus, the equation of the curve is:

$y = x^{1/k} c$

Question36

The semi-vertical angle of a right circular cone is $45°\%$ If the radius of the base of the cone is measured as 14 cm with an error of $\left(\frac{\sqrt{2}-1}{11}\right)\mathrm{cm}$, then the approximate error in measuring its total surface area is (in sq cm)

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Options:

A. 14

B. 8

C. 5

D. 3

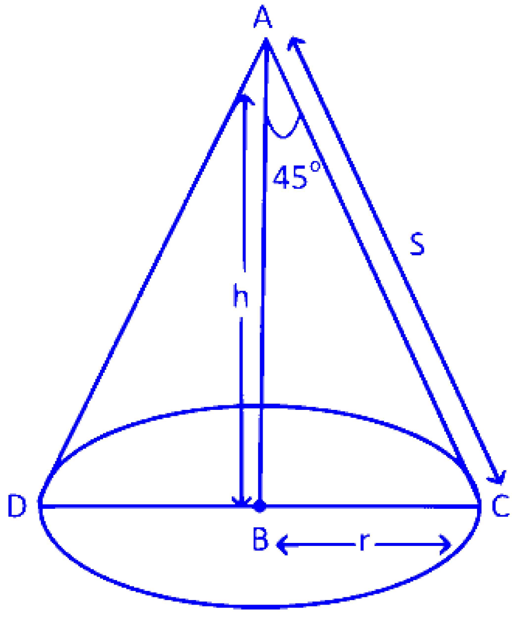
Answer: B

Solution:

Given that

Semi-vertical angle of a right circular cone $= 45^{\circ}$

Radius of base of the cone $= 14$ cm with an error $(\sqrt{2}-1)/11$



Total surface area $= \pi r^2 + \pi r s$

$$\text{Error } = \frac{dr}{r} = \frac{\sqrt{2}-1}{11}$$

$$\Rightarrow \quad dr = r \times \frac{\sqrt{2}-1}{11} = \frac{14}{11}(\sqrt{2}-1)$$

From $\triangle ABC, \tan 45^\circ = \frac{r}{h} \Rightarrow r = h$

$$\Rightarrow \quad h = 14 \text{ cm} \Rightarrow s^2 = h^2 + r^2 = 2r^2$$

$$\Rightarrow \quad s = \sqrt{2} \cdot r = 14\sqrt{2}$$

Since, error in total surface area

$ds = \frac{\partial s}{\partial r} dr + \frac{\partial s}{\partial h} \cdot dh$

After simplifying

$ds = 8$

Question37

If a man of height 1.8 mt , is walking away from the foot of a light pole of height 6 mt , with a speed of 7 km per hour on a straight horizontal road opposite to the pole, then the rate of change of the length of his shadow is (in kmph)

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Options:

A. 7

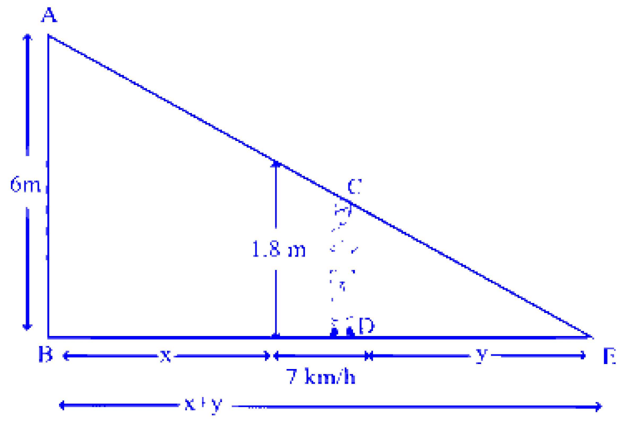
B. 5

C. 3

D. 2

Answer: C

Solution:



$\triangle ABE$ and $\triangle CDE$ are similar

$$\frac{AB}{CD} = \frac{BE}{DE}$$

$$\Rightarrow \quad \frac{6}{1 \cdot 8} = \frac{x+y}{y} = 6y = 1 \cdot 8x + 1 \cdot 8y$$

$$\Rightarrow \quad 7y = 3x$$

On differentiating both sides w.r to t, we get

$3 \frac{dx}{dt} = 7 \frac{dy}{dt}$

$\therefore$ Main velocity

$\Rightarrow \quad \frac{dy}{dt} = \frac{3 \times 7}{7} = 3$

Question38

If the curves $2x^2 + ky^2 = 30$ and $3y^2 = 28x$ cut each other orthogonally, then k is equal to

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Options:

A. 5

B. 3

C. 2

D. 1

Answer: D

Solution:

Given that

$C_1 : 2x^2 + ky^2 = 30$

and $C_2 : 3y^2 = 28x$ cut each other orthogonaly.

$$\Rightarrow \quad 2x^2 + ky^2 = 30$$
$$\Rightarrow \quad 4x + 2ky \cdot \frac{dy}{dx} = 0$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{-4x}{2ky} = -\frac{2x}{ky}$$
$$\therefore \quad m_1 = -\frac{2x}{ky}$$

$$Now, \; 3y^2 = 28x$$
$$6y \cdot \frac{dy}{dx} = 28 \Rightarrow \frac{dy}{dx} = \frac{28}{6y} = \frac{14}{3y}$$
$$\therefore \quad m_2 = \frac{14}{3y}$$

For orthogonally

$m_1 m_2 = -1$

$$\Rightarrow \quad -\frac{2x}{ky} \cdot \frac{14}{3y} = -1, \quad k = \frac{3y^2}{28x}$$

Now, find intersection point of both curve

$$\frac{y^2}{x} = \frac{28}{3} \quad [\text{ from curve } C_2]$$

$$\Rightarrow \quad K = \frac{3}{28} \times \frac{28}{3} = 1$$

$$K = 1$$

Question39

The interval containing all the real values of x such that the real valued function $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$ is strictly increasing is

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Options:

A. $(1, \infty)$

B. $(0, 1)$

C. $(-\infty, 0) \cup (1, \infty)$

D. $(-\infty, 0)$

Answer: A

Solution:

We have, $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$

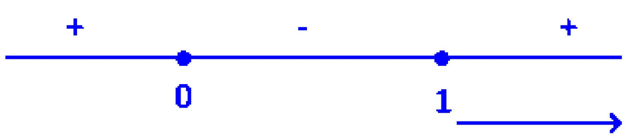
Interval in which $f(x)$ strictly increasing $f'(x) > 0$.

$$\frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) > 0$$

$$\Rightarrow \frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}} > 0 \Rightarrow \frac{1}{\sqrt{x}} > \frac{1}{x \cdot x^{1/2}}$$

$$\Rightarrow \quad x\sqrt{x} > \sqrt{x} \Rightarrow \sqrt{x}(x-1) > 0$$

$$\Rightarrow \quad \sqrt{x}(x-1) > 0$$



$\therefore \quad x \in (1, \infty)$

Question40

The value of Lagrange's mean value theorem for $f(x) = e^x + 24$ in $[0, 1]$ is

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Options:

A. $\log(e-1)$

B. $\log(e+1)$

C. $\log(e+24)$

D. $\log(e-24)$

Answer: A

Solution:

Let's address the value of Lagrange's Mean Value Theorem for the function $f(x) = e^x + 24$ over the interval $[0, 1]$.

According to the theorem, if a function f is continuous on $[a, b]$ and differentiable on (a, b), then there exists at least one point c such that:

$f'(c) = \frac{f(b)-f(a)}{b-a}$

For this specific function:

$f(x) = e^x + 24$

Interval: $[0, 1]$

Calculate $f(0)$ and $f(1)$:

$f(0) = e^0 + 24 = 1 + 24 = 25$

$f(1) = e^1 + 24 = e + 24$

The derivative, $f'(x)$, is:

$f'(x) = e^x$

Next, let's find the average rate of change over the interval $[0, 1]$:

$\frac{f(1)-f(0)}{1-0} = \frac{(e+24)-25}{1} = e - 1$

According to Lagrange's Theorem, set:

$f'(c) =$ Average rate of change

Thus, we have:

$e^c = e - 1$

To find c, solve for c:

$c = \log(e - 1)$

This yields the point $c = \log(e - 1)$ as the value of the theorem for the function over the given interval.

Question41

Equation of the normal to the curve $y = x^2 + x$ at the point $(1, 2)$ is

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Options:

A. $x - 3y + 5 = 0$

B. $x + 3y + 5 = 0$

C. $x + 3y + 7 = 0$

D. $x + 3y - 7 = 0$

Answer: D

Solution:

Given the curve $y = x^2 + x$ and the point $(1, 2)$, we need to find the equation of the normal to the curve at this point.

First, find the derivative of the curve to get the slope of the tangent:

$\frac{dy}{dx} = 2x + 1$

Calculate the slope at the point $(1, 2)$:

$\left.\frac{dy}{dx}\right|_{(1,2)} = 2 \times 1 + 1 = 3$

The slope of the tangent line at $(1, 2)$ is 3. The slope of the normal line is the negative reciprocal of this, which is:

$-\frac{1}{3}$

Using the point-slope form of the equation of a line, the equation of the normal line is:

$y - y_1 = m(x - x_1)$

Here, $x_1 = 1$, $y_1 = 2$, and the slope $m = -\frac{1}{3}$.

Substitute the values into the equation:

$y - 2 = -\frac{1}{3}(x - 1)$

Simplify:

$3(y - 2) = -(x - 1)$

$3y - 6 = -x + 1$

Rearrange to standard form:

$x + 3y - 7 = 0$

Thus, the equation of the normal to the curve at the point $(1, 2)$ is:

$x + 3y - 7 = 0$

Question42

Displacement s of a particle at time t is expressed as $s = 2t^3 - 9t$. Find the acceleration at the time when b^{t5} velocity vanishes.

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Options:

A. 6

B. $6\sqrt{3}$

C. $6\sqrt{6}$

D. $3\sqrt{6}$

Answer: C

Solution:

The position of a particle, given by its displacement s at time t, is expressed as $s = 2t^3 - 9t$.

To find velocity v, we differentiate the displacement with respect to time t:

$v = \frac{d}{dt}(2t^3 - 9t) = 6t^2 - 9$

The velocity vanishes (i.e., $v = 0$) at:

$6t^2 - 9 = 0$

Solving for t, we get:

$t^2 = \frac{9}{6} = \frac{3}{2}$

$t = \sqrt{\frac{3}{2}}$

Next, to find the acceleration a, we differentiate the velocity:

$a = \frac{d}{dt}(6t^2 - 9) = 12t$

Substituting $t = \sqrt{\frac{3}{2}}$ into the acceleration equation, we find:

$a = 12 \times \sqrt{\frac{3}{2}}$

Simplifying further, we get:

$a = 12 \times \frac{\sqrt{6}}{\sqrt{2}}$

Thus:

$a = 6\sqrt{6}$

Therefore, the acceleration at the time when the velocity vanishes is $6\sqrt{6}$.

Question43

If a running track of 500 ft is to be laid out enclosing a playground the shape of which is a rectangle with a semi-circle at each end, then the length of the rectangular portion such that the area of the rectangular portion is to be maximum is (in feet)

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Options:

A. 100

B. 150

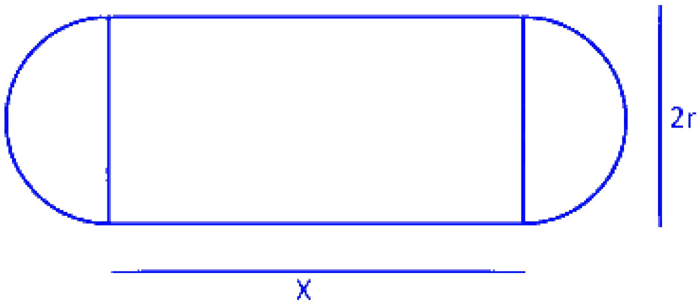
C. 125

D. 200

Answer: C

Solution:

Area of rectangle point $= 2xx$



$$\begin{aligned} &\text{track } = 500 \\ \therefore\ & 500 = 2x + 2\pi r \\ & 250 = x + \pi r \end{aligned}$$

So, area of rectangle point $f(r)$

$= 2r(250 - \pi r)$

Area to be maximum $f'(r) = 0$

$$\Rightarrow 500 - 4\pi r$$
$$\Rightarrow r = \frac{125}{\pi}$$

So, length of rectangle portion is 125 .

Question44

If x is real and α, β are maximum and minimum values of $\frac{x^2-x+1}{x^2+x+1}$ respectively, then $\alpha + \beta =$

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Options:

A. $\frac{10}{3}$

B. $\frac{8}{3}$

C. $\frac{4}{3}$

D. $\frac{2}{3}$

Answer: A

Solution:

Given that x is real and α, β are the maximum and minimum values of $\frac{x^2-x+1}{x^2+x+1}$, respectively.

$$\text{Let} \qquad y = \frac{x^2-x+1}{x^2+x+1}$$

$$\Rightarrow yx^2 + yx + y = x^2 - x + 1$$

$$\Rightarrow (y-1)x^2 + (y+1)x + y - 1 = 0$$

$$(x \in R)$$

$$\begin{aligned} &\text{Consider } b^2 - 4ac \geq 0 \\ &\Rightarrow \quad (y+1)^2 - 4(y-1)^2 \geq 0 \\ &\Rightarrow (y+1+2y-2)(y+1-2y+2) \geq 0 \\ &\Rightarrow \quad (3y-1)(y-3) \leq 0 \end{aligned}$$

$$\text{Range } = \left[\frac{1}{3}, 3\right]$$

$$\because \text{ Minimum value } = \frac{1}{3} = \beta$$

$$\text{Maximum value } = 3 = \alpha$$

$$\alpha + \beta = 3 + \frac{1}{3} = \frac{10}{3}$$

Question45

The value of c such that the straight line joining the points $(0, 3)$ and $(5, -2)$ is tangent to the curve $y = \frac{c}{x+1}$ is

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Options:

A. 3

B. 4

C. 5

D. 2

Answer: B

Solution:

To find the value of c such that the line joining the points $(0, 3)$ and $(5, -2)$ is tangent to the curve $y = \frac{c}{x+1}$, follow these steps:

Equation of the Curve:

The given equation of the curve is:

$y = \frac{c}{x+1}$

Differentiate the Curve:

Differentiate the curve equation with respect to x:

$\frac{dy}{dx} = \frac{-c}{(x+1)^2}$

This derivative gives the slope of the tangent to the curve.

Find Slope of the Tangent Line:

Calculate the slope of the line joining the points $(0, 3)$ and $(5, -2)$:

Slope $= \frac{-2-3}{5-0} = -1$

The equation of the line, therefore, is:

$y = -x + 3$

Set the Slopes Equal:

Since the line is tangent to the curve, set the slopes equal:

$-\frac{c}{(x+1)^2} = -1$

Simplifying gives:

$c = (x+1)^2$

Equation of the Curve at Tangency Point:

For the tangency condition, equate the line and the curve:

$-x + 3 = \frac{c}{x+1}$

Substitute $c = (x+1)^2$ into this equation:

$-x + 3 = x + 1$

Solve for x:

$2x = 2$

$x = 1$

Calculate c:

Substitute $x = 1$ back into $c = (x+1)^2$:

$c = (1+1)^2 = 4$

Thus, the value of c is 4.

Question46

If the percentage error in the radius of circle is 3 , then the percentage error in its area is

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Options:

A. 6

B. 3/2

C. 2

D. 4

Answer: A

Solution:

If the percentage error in the radius of a circle is 3%, we can calculate the percentage error in its area as follows:

The formula for the percentage error in the radius is:

$\frac{\delta r}{r} \times 100 = 3$

The area A of a circle is given by:

$A = \pi r^2$

Taking the logarithm of both sides of the area formula gives:

$\log A = \log(\pi r^2)$

This can be expanded to:

$\log A = \log \pi + 2 \log r$

Differentiating both sides with respect to the logarithmic variables:

$\frac{1}{A} \cdot \delta A = 0 + 2 \cdot \frac{1}{r} \delta r$

Thus:

$\frac{\delta A}{A} = \frac{2\delta r}{r}$

Multiplying both sides by 100 to find the percentage error in the area:

$100 \times \frac{\delta A}{A} = 2 \times \frac{\delta r}{r} \times 100$

Substituting the known percentage error of the radius from Equation (i):

$100 \times \frac{\delta A}{A} = 2 \times 3$

Simplifying gives:

$100 \times \frac{\delta A}{A} = 6$

Therefore, the percentage error in the area is 6%.

Question47

The equation of the tangent to the curve $y = x^3 - 2x + 7$ at the point $(1, 6)$ is

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Options:

A. $y = x + 5$

B. $x + y = 7$

C. $2x + y = 8$

D. $x + 2y = 13$

Answer: A

Solution:

To find the equation of the tangent to the curve $y = x^3 - 2x + 7$ at the point $(1, 6)$, we need to follow these steps:

Find the derivative:

The derivative of the function $y = x^3 - 2x + 7$ with respect to x is given by:

$\frac{dy}{dx} = 3x^2 - 2$

Determine the slope of the tangent line:

Calculate the slope (m) of the tangent line at the point $(1, 6)$ using the derivative:

$m = \frac{dy}{dx}\Big|_{x=1} = 3(1)^2 - 2 = 1$

Write the equation of the tangent line:

Using the point-slope form of a line equation, $y - y_1 = m(x - x_1)$, where $(x_1, y_1) = (1, 6)$, we substitute $m = 1$:

$y - 6 = 1(x - 1)$

Simplify the equation:

Simplifying the equation gives:

$y - 6 = x - 1$

$y = x + 5$

Thus, the equation of the tangent to the curve at the point $(1, 6)$ is $y = x + 5$.

Question48

The distance (s) travelled by a particle in time t is given by $S = 4t^2 + 2t + 3$. The velocity of the particle, when $t = 3\text{sec}$ is

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Options:

A. 26 unit/sec

B. 20 unit/sec

C. 24 unit /sec

D. 30 unit/sec

Answer: A

Solution:

The distance S travelled by a particle over time t is given by the equation:

$S = 4t^2 + 2t + 3 \qquad \ldots(i)$

To find the velocity of the particle at $t = 3$ seconds, we need to differentiate equation (i) with respect to time t. Velocity is defined as the derivative of distance with respect to time. Therefore, we have:

Velocity $= \frac{\text{Distance}}{\text{Time}}$

$\Rightarrow v = \frac{dS}{dt} = \frac{d}{dt}(4t^2 + 2t + 3)$

When we differentiate the expression, we get:

$v = \frac{dS}{dt} = (8t + 2)$ units/sec

Substituting $t = 3$ into the velocity equation, we calculate:

$V = (8 \times 3 + 2)$ units/sec

$V = (24 + 2)$ units/sec

Thus, the velocity V at $t = 3$ seconds is:

$V = 26$ units/sec

Question49

If $a^2x^4 + b^2y^4 = c^6$, then maximum value of xy is equal to

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Options:

A. $\frac{c^3}{2ab}$

B. $\frac{c^3}{\sqrt{2ab}}$

C. $\frac{c^3}{ab}$

D. $\frac{c^3}{\sqrt{ab}}$

Answer: B

Solution:

We start with the equation:

$a^2x^4 + b^2y^4 = c^6$

This can be rearranged to express y^4:

$y^4 = \frac{c^6 - a^2x^4}{b^2}$

Let $z = xy$. To find the maximum value of z, we express z in terms of x and differentiate:

$z = x\left(\frac{c^6 - a^2x^4}{b^2}\right)^{1/4}$

Differentiate z with respect to x:

$\frac{dz}{dx} = \left(\frac{c^6 - a^2x^4}{b^2}\right)^{1/4} - \frac{x}{4}\left(\frac{c^6 - a^2x^4}{b^2}\right)^{-3/4} \cdot 4x^3 \frac{a^2}{b^2}$

Simplify:

$z'(x) = \left(\frac{c^6 - a^2x^4}{b^2}\right)^{1/4} - \frac{x^4a^2}{b^2}\left(\frac{c^6 - a^2x^4}{b^2}\right)^{-3/4}$

Set $z'(x) = 0$ to find critical points:

$\left(\frac{c^6 - a^2x^4}{b^2}\right)^{1/4} = \frac{x^4a^2}{b^2}\left(\frac{c^6 - a^2x^4}{b^2}\right)^{-1}$

$x^4 = \frac{b^2}{a^2}\left(\frac{c^6 - a^2x^4}{b^2}\right)$

$x^4 = \frac{c^6 - a^2x^4}{a^2}$

This results in:

$x^4 = \frac{c^6}{2a^2}$

Thus, solving for x:

$x = \left(\frac{c^6}{2a^2}\right)^{1/4} = \frac{c^{6/4}}{(2^{1/4}a^{2/4})}$

Now find $z = xy$:

$z = \frac{c^{6/4}}{2^{1/4}a^{1/2}}\left(\frac{c^6 - a^2x^4}{b^2}\right)^{1/4}$

Substitute $x^4 = \frac{c^6}{2a^2}$ back into the equation:

$z = \frac{c^{6/4}}{2^{1/4}a^{1/2}}\left(\frac{c^6}{2b^2}\right)^{1/4}$

Finally, evaluate z:

$z = \left(\frac{c^{12}}{4a^2b^2}\right)^{1/4} = \frac{c^3}{\sqrt{2ab}}$

The maximum value of xy is:

$\frac{c^3}{\sqrt{2ab}}$

Question50

If a number is drawn at random from the set $\{1, 3, 5, 7, ...59\}$, then the probability that it lies in the interval in which the function

$f(x) = x^3 - 16x^2 + 20x - 5$ **is stricly decreasing is**

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Options:

A. $\frac{1}{5}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{1}{6}$

Answer: D

Solution:

We have the function $f(x) = x^3 - 16x^2 + 20x - 5$.

Step 1: Find the Critical Points

First, find the derivative of $f(x)$:

$f'(x) = 3x^2 - 32x + 20$

Set the derivative equal to zero to find critical points:

$3x^2 - 32x + 20 = 0$

Solving this quadratic equation, we find:

$x = \frac{32 \pm \sqrt{1024 - 240}}{6} = \frac{32 \pm \sqrt{784}}{6}$

Simplifying, we get:

$x = \frac{32 \pm 28}{6}$

Thus, the roots are $x = 10$ and $x = \frac{2}{3}$.

Step 2: Identify Intervals of Decrease

The function is strictly decreasing in the interval $(\frac{2}{3}, 10)$.

Step 3: Analyze the Number Set

The set of numbers is $\{1, 3, 5, 7, 9, \ldots, 59\}$.

Step 4: Count Favourable Outcomes

Within the decreasing interval $(\frac{2}{3}, 10)$, the numbers from the set are $\{1, 3, 5, 7, 9\}$. Hence, there are 5 favorable outcomes.

Step 5: Count Total Outcomes

The sequence $1, 3, 5, 7, \ldots, 59$ is an arithmetic progression (AP) where:

First term $a = 1$

Common difference $d = 2$

Last term $a_n = 59$

To find the number of terms n, use the formula:

$a_n = a + (n - 1)d$

$59 = 1 + (n - 1) \times 2$

$n = 30$

So, the total number of outcomes is 30.

Step 6: Calculate Probability

The probability P is given by:

$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{5}{30} = \frac{1}{6}$

Thus, the probability that a randomly drawn number from the set $\{1, 3, 5, 7, \ldots, 59\}$ lies in the interval where the function $f(x) = x^3 - 16x^2 + 20x - 5$ is strictly decreasing is $\frac{1}{6}$.

Question51

The equation of the normal drawn to the parabola $y^2 = 6x$ at the point $(24, 12)$ is

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Options:

A. $3x - y = 60$

B. $4x + y = 108$

C. $2x + y = 60$

D. $x - 2y = 0$

Answer: B

Solution:

To find the equation of the normal line to the parabola given by $y^2 = 6x$ at the point $(24, 12)$, follow these steps:

Determine the Slope of the Tangent Line:

The derivative of the parabola's equation $y^2 = 6x$ with respect to x is:

$$2y\frac{dy}{dx} = 6 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{3}{y}$$

Substitute $y = 12$ to find the slope at the point $(24, 12)$:

$$\left[\frac{dy}{dx}\right]_{(24,12)} = \frac{3}{12} = \frac{1}{4}$$

Find the Equation of the Normal Line:

The slope of the normal line is the negative reciprocal of the tangent line's slope. Thus, the slope of the normal line is:

$$m = -\frac{1}{\frac{1}{4}} = -4$$

Use the point-slope form of a line equation for the normal line at $(24, 12)$:

$$y - y_1 = m(x - x_1)$$

$$y - 12 = -4(x - 24)$$

Simplify and rearrange the equation:

$$y - 12 = -4x + 96$$

$$4x + y = 108$$

Thus, the equation of the normal line to the parabola $y^2 = 6x$ at the point $(24, 12)$ is:

$$4x + y = 108$$

Question52

The point which lies on the tangent drawn to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point $(1, 0)$ is

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Options:

A. $(2, 6)$

B. $(2, -6)$

C. $(-2, -6)$

D. $(-2, 6)$

Answer: D

Solution:

To find the point that lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point $(1, 0)$, we begin by differentiating the equation with respect to x:

$x^4e^yy' + e^y4x^3 + \frac{2y'}{2\sqrt{y+1}} = 0$

Evaluating the derivative at the point $P(1, 0)$:

$1^4e^0y_p' + e^0 \cdot 4 \cdot 1^3 + \frac{2y_p'}{2\sqrt{0+1}} = 0$

$y_p' + 4 + y_p' = 0$

$\Rightarrow 2y_p' = -4$

$\Rightarrow y_p' = -2$

Hence, the equation of the tangent at $P(1, 0)$ is:

$y - 0 = -2(x - 1)$

Simplifying, we get:

$2x + y = 2$

Now substituting $x = -2$ into the tangent equation:

$2(-2) + y = 2$

$-4 + y = 2$

$y = 6$

Thus, the point $(-2, 6)$ lies on the tangent line.

Question53

If $f(x) = x^x$, then the interval in which $f(x)$ decrease is

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Options:

A. $\left[0, \frac{1}{e}\right]$

B. $[0, \mathrm{e}]$

C. $\left[\frac{1}{\theta}, \infty\right]$

D. $\left[0, e^{e}\right]$

Answer: A

Solution:

To determine the interval where the function $f(x)=x^{x}$ decreases, we first express it as:

$f(x)=x^{x}=e^{x \ln x}$

To find when the function decreases, we differentiate it. Using the chain rule:

$\frac{d}{d x}\left[e^{x \ln x}\right]=e^{x \ln x} \cdot \frac{d}{d x}[x \ln x]$

Next, we calculate the derivative of $x \ln x$:

$\frac{d}{d x}[x \ln x]=\ln x+1$

Therefore, the derivative $f^{\prime}(x)$ of the function can be written as:

$f^{\prime}(x)=e^{x \ln x}(\ln x+1)=x^{x}(\ln x+1)$

The function $f(x)$ decreases when $f^{\prime}(x)$ is negative:

$x^{x}(\ln x+1) \leq 0$

Since $x^{x} \geq 0$ for $x \geq 0$, this implies:

$\ln x+1<0$

Solving for x, we find:

$\ln x \leq-1$

$x<\frac{1}{e}$

Thus, the interval where $f(x)=x^{x}$ is decreasing is:

$\left[0, \frac{1}{e}\right]$

Question54

If the Rolle's theorem is applicable for the function $f(x)$ defined by $f(x)=x^{3}+P x-12$ on $[0,1]$ then the value of C of the Rolle's theorem is

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Options:

A. $\pm\frac{1}{\sqrt{3}}$

B. $-\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{3}$

Answer: C

Solution:

To apply Rolle's Theorem to the function $f(x) = x^3 + Px - 12$ on the interval $[0, 1]$, we must first ensure that the function satisfies the conditions of the theorem: $f(x)$ must be continuous on $[0, 1]$, differentiable on $(0, 1)$, and $f(0) = f(1)$.

Firstly, let's verify that $f(0) = f(1)$:

$f(0) = 0^3 + P \cdot 0 - 12 = -12$

$f(1) = 1^3 + P \cdot 1 - 12 = 1 + P - 12 = P - 11$

Setting $f(0) = f(1)$, we get:

$-12 = P - 11 \quad \Rightarrow \quad P = -1$

With $P = -1$, the function becomes:

$f(x) = x^3 - x - 12$

We then find $f'(x)$ and check for points where $f'(c) = 0$ in the interval $(0, 1)$:

$f'(x) = \frac{d}{dx}[x^3 - x - 12] = 3x^2 - 1$

Setting $f'(c) = 0$:

$3c^2 - 1 = 0 \quad \Rightarrow \quad c^2 = \frac{1}{3} \quad \Rightarrow \quad c = \pm\frac{1}{\sqrt{3}}$

Since c must be in the interval $(0, 1)$, we choose the positive value:

$c = \frac{1}{\sqrt{3}}$

Thus, for the function $f(x)$ under the conditions of Rolle's Theorem, $c = \frac{1}{\sqrt{3}}$ is the appropriate value within the interval $(0, 1)$.

Question55

The number of all the value of x for which the function $f(x) = \sin x + \frac{1-\tan^2 x}{1+\tan^2 x}$ attains it maximum value on [0.2π] is

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Options:

A. 4

B. 1

C. 2

D. infinite

Answer: C

Solution:

The function given is:

$f(x) = \sin x + \frac{1-\tan^2 x}{1+\tan^2 x}$

Recall the trigonometric identity:

$\frac{1-\tan^2 x}{1+\tan^2 x} = \cos 2x$

Substituting, we get:

$f(x) = \sin x + \cos 2x$

To find the maximum value of this function, we first compute its derivative:

$f'(x) = \cos x - 4 \sin x \cos x = \cos x(1 - 4 \sin x)$

Setting the derivative equal to zero gives us:

$\cos x = 0$

This occurs when:

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$1 - 4 \sin x = 0$

Solving for $\sin x$, we get:

$\sin x = \frac{1}{4}$

So:

$x = \sin^{-1} \frac{1}{4}, \quad x = \pi - \sin^{-1} \frac{1}{4}$

Thus, the critical points are:

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \sin^{-1} \frac{1}{4}, \pi - \sin^{-1} \frac{1}{4}$

Now evaluate the function at these critical points:

$f\left(\frac{\pi}{2}\right) = 0$

$f\left(\frac{3\pi}{2}\right) = -2$

For:

$f\left(\sin^{-1} \frac{1}{4}\right)$

$f\left(\pi - \sin^{-1} \frac{1}{4}\right)$

These values are where the function attains its maximum. The maximum value is 2.

Question56

Equation of a tagent line of the parabola $y^2 = 8x$, which passes through the point $(1, 3)$ is

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Options:

A. $y = 2x + 1$

B. $2y = x + 5$

C. $y = -2x + 5$

D. $2y = 3x + 3$

Answer: A

Solution:

Given the parabola:

$y^2 = 8x$

Rewriting the equation, we have:

$x = \frac{y^2}{8} \quad ...(\text{i})$

Differentiating both sides with respect to y, we find:

$\frac{dx}{dy} = \frac{2y}{8} = \frac{y}{4}$

Thus, the slope (m) of the tangent line is $m = \frac{y}{4}$.

The equation of a tangent line passing through the point $(1, 3)$ is:

$(x - 1) = \frac{y}{4}(y - 3) \quad ...(\text{ii})$

Combining equations (i) and (ii), we have:

$$\frac{y^2}{8} - 1 = \frac{y}{4}(y - 3)$$
$$\Rightarrow y^2 - 8 = 2y^2 - 6y$$
$$\Rightarrow y^2 - 6y + 8 = 0$$
$$\Rightarrow (y - 4)(y - 2) = 0$$
$$\therefore y = 4 \text{ and } y = 2$$

Using equation (i) to find x:

For $y = 4$:

$x = \frac{4^2}{8} = 2$

For $y = 2$:

$x = \frac{2^2}{8} = \frac{1}{2}$

Thus, the points are $(2, 4)$ and $\left(\frac{1}{2}, 2\right)$.

The required equation of the tangent line to the parabola $y^2 = 8x$, which passes through the points $\left(\frac{1}{2}, 2\right)$ and $(1, 3)$, is found as follows:

$$(y - 3) = \frac{(3 - 2)}{\left(1 - \frac{1}{2}\right)}(x - 1)$$
$$\Rightarrow y - 3 = \frac{1}{\frac{1}{2}}(x - 1) = 2x - 2$$
$$\Rightarrow y = 2x - 2 + 3 = 2x + 1$$

Thus, the equation of the tangent line is:

$y = 2x + 1$

Question57

p_1 and p_2 are the perpendicular distances from the origin to the tangent and normal drawn at any point on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

respectively. If $k_1 p_1^2 + k_2 p_2^2 = a^2$, then $k_1 + k_2 =$

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Options:

A. 7

B. 6

C. 5

D. 4

Answer: C

Solution:

Given curve

$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}} \quad \ldots \text{(i)}$

Let consider the point $\left(a \cos^3 \theta, a \sin^3 \theta\right)$

Now, slope of the tangent $= -\tan \theta$

$\therefore$ Equation of tangent is

$y - a \sin^3 \theta = -\tan \theta \left(x - a \cos^3 \theta\right)$

$\Rightarrow x \sin \theta + y \cos \theta = a \sin \theta \cos \theta$

$\Rightarrow x \sin \theta + y \cos \theta = \frac{a}{2} \sin 2\theta \quad \ldots \text{(ii)}$

Slope of normal $= \cot \theta$

$\therefore$ Equation of normal is $\left(y - a \sin^3 \theta\right) = \cot \theta \left(x - a \cos^3 \theta\right)$

$\Rightarrow x \cos \theta - y \sin \theta = a \cos 2\theta \quad \ldots \text{(iii)}$

Now, $P_1 = \left|\frac{-a \sin \theta \cos \theta}{\sqrt{\sin^2 \theta + \cos^2 \theta}}\right| = \frac{a}{2} \sin 2\theta$

$\Rightarrow \quad 4P_1^2 = a^2 \sin^2 2\theta \quad \ldots \text{(iv)}$

and $P_2 = \left|\frac{-a \cos 2\theta}{\sqrt{\sin^2 \theta + \cos^2 \theta}}\right| = a \cos 2\theta$

$\Rightarrow P_2^2 = a^2 \cos^2 2\theta$

On adding Eqs. (i) and (ii), we get

$$4P_1^2 + P_2^2 = a^2\left(\sin^2 2\theta + \cos^2 2\theta\right)$$
$$= a^2 = k_1P_1^2 + k_2P_2^2 \quad \text{[given]}$$

On comparing, we get

$$k_1 = 4, k_2 = 1$$
$$\therefore k_1 + k_2 = 4 + 1 = 5$$

Question58

The length of the subnormal at any point on the curve $y = \left(\frac{x}{2024}\right)^k$ is constant, if the value of k is

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Options:

A. 1

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. 0

Answer: C

Solution:

Given the curve:

$y = \left(\frac{x}{2024}\right)^k$

Let $P(x_1, y_1)$ be a point on this curve. Then:

$y_1 = \left(\frac{x_1}{2024}\right)^k$

Differentiating Eq. (i) with respect to x, we have:

$\frac{dy}{dx} = k\left(\frac{x}{2024}\right)^{k-1} \cdot \frac{1}{2024}$

Thus, the slope m at the point (x_1, y_1) is:

$m = \frac{k}{2024}\left(\frac{x_1}{2024}\right)^{k-1}$

The length of the subnormal at point P is given by:

$|y_1 m| = \left|\left(\frac{x_1}{2024}\right)^k \cdot \frac{k}{2024}\left(\frac{x_1}{2024}\right)^{k-1}\right| = \frac{k}{2024}\left(\frac{x_1}{2024}\right)^{2k-1}$

Since the length of the subnormal is constant, it must be independent of x_1. Thus, we have:

$2k - 1 = 0$

Solving this equation gives:

$k = \frac{1}{2}$

Question59

The acute angle between the curves $x^2 + y^2 = x + y$ and $x^2 + y^2 = 2y$ is

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Options:

A. $\frac{2\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer: D

Solution:

Given, curve $x^2 + y^2 = x + y \quad \ldots$ (i)

and $x^2 + y^2 = 2y \quad \ldots$ (ii)

On differentiate Eq. (i) w.r.t. x, we get

$$\begin{aligned} & 2x + 2y\frac{dy}{dx} = 1 + \frac{dy}{dx} \\ \Rightarrow \quad & (2y-1)\frac{dy}{dx} = 1 - 2x \\ \Rightarrow \quad & m_1 = \frac{dy}{dx} = \frac{1-2x}{2y-1} \end{aligned}$$

Again, differentiate Eq. (ii) w.r.t. x, we get

$$\begin{aligned} & 2x + 2y\frac{dy}{dx} = 2\frac{dy}{dx} \\ & (y-1)\frac{dy}{dx} = -x \\ \Rightarrow \quad & m_2 = \frac{dy}{dx} = \frac{x}{1-y} \end{aligned}$$

From Eqs. (i) and (ii), we get

$x+y=2y \Rightarrow x=y$

From Eq. (i), $2x^2=2x$

$\Rightarrow \quad x=0,1 \Rightarrow y=0,1$
At $(0,0)$

$$m_1=\frac{1-0}{0-1}=-1$$
$$m_2=\frac{0}{1-0}=0$$

So, $\tan \theta=\left|\frac{m_1+m_2}{1-m_1 m_2}\right|=\left|\frac{-1+0}{1-0}\right|=1$

$\Rightarrow \quad \theta=\frac{\pi}{4}$

Question60

A' value of C according to the Lagrange's mean value theorem for $f(x)=(x-1)(x-2)(x-3)$ in $[0,4]$ is

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Options:

A. $2+\frac{2}{\sqrt{3}}$

B. $2-\sqrt{\frac{16}{3}}$

C. $1+\sqrt{\frac{5}{4}}$

D. $2+\sqrt{\frac{8}{3}}$

Answer: A

Solution:

Given the function $f(x)=(x-1)(x-2)(x-3)$, we know that it is a cubic polynomial. This ensures that it is continuous and differentiable in the interval $[0,4]$.

Start by expanding the function:

$f(x)=(x-1)(x-2)(x-3)$

This expands to:

$f(x) = x^3 - 6x^2 + 11x - 6$

Next, find the derivative $f'(x)$:

$f'(x) = 3x^2 - 12x + 11$

Lagrange's Mean Value Theorem states that there exists some c in the interval $(0, 4)$ such that:

$f'(c) = \frac{f(4)-f(0)}{4-0}$

Calculate $f(4)$ and $f(0)$:

$f(4) = (4-1)(4-2)(4-3) = 3 \times 2 \times 1 = 6$

$f(0) = (0-1)(0-2)(0-3) = (-1) \times (-2) \times (-3) = -6$

Thus,

$f'(c) = \frac{6-(-6)}{4} = \frac{12}{4} = 3$

Set the derivative equal to this value:

$3c^2 - 12c + 11 = 3$

Rearrange to form a quadratic equation:

$3c^2 - 12c + 8 = 0$

Now, solve this quadratic equation using the quadratic formula:

$c = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

In this equation, $a = 3$, $b = -12$, and $c = 8$. Thus:

$c = \frac{12 \pm \sqrt{(-12)^2-4 \cdot 3 \cdot 8}}{6}$

$c = \frac{12 \pm \sqrt{144-96}}{6}$

$c = \frac{12 \pm \sqrt{48}}{6}$

$c = \frac{12 \pm 4\sqrt{3}}{6}$

$c = 2 \pm \frac{2}{\sqrt{3}}$

Since both roots $c = 2 + \frac{2}{\sqrt{3}}$ and $c = 2 - \frac{2}{\sqrt{3}}$ are valid within the interval $(0, 4)$, any of these can be a valid 'c' in the context of Lagrange's Mean Value Theorem.

Question61

If $T = 2\pi\sqrt{\frac{L}{g}}$, g is a constant and the relative error in T is k times to the percentage error in l, then $\frac{1}{K}$ =

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Options:

A. 2

B. $\frac{1}{200}$

C. 200

D. $\frac{1}{2}$

Answer: C

Solution:

We start with the equation for the period T:

$T = 2\pi\sqrt{\frac{L}{g}}$

Given that g is constant, the derivative of T with respect to L is:

$\frac{dT}{dL} = \frac{\pi}{\sqrt{Lg}}$

The approximate error in T is:

$dT = \frac{dT}{dL} \times \Delta L = \frac{\pi}{\sqrt{Lg}} \times \Delta L$

The relative error in T is:

$\frac{dT}{T} = \frac{1}{2L}\Delta L$

According to the problem, the relative error in T is K times the percentage error in L, which is expressed as:

$\frac{\Delta T}{T} = K\frac{\Delta L}{L} \times 100$

Substituting from the relative error expression, we have:

$\frac{1}{2L}\Delta L = K\frac{\Delta L}{L} \times 100$

Solving for K, we find:

$K = \frac{1}{200}$

Thus, the reciprocal of K is:

$\frac{1}{K} = 200$

Question62

The angle between the curves $y^2 = 2x$ and $x^2 + y^2 = 8$ is

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Options:

A. $\tan^{-1}(1)$

B. $\tan^{-1}(2)$

C. $\tan^{-1}(3)$

D. $\tan^{-1}\left(-\frac{1}{2}\right)$(d) $\tan^{-1}\left(-\frac{1}{2}\right)$

Answer: C

Solution:

We have,

$$\begin{aligned}&y^2 = -2x \text{ and } x^2 + y^2 = 8\\ \Rightarrow\quad &x^2 + 2x - 8 = 0\\ &x^2 + 4x - 2x - 8 = 0\\ &(x+4)(x-2) = 0\\ &x = -4 \text{ or } x = 2\end{aligned}$$

If $x = 2$, then $y = \pm 2$ and $x \neq -4$ because $y^2 \neq -8$

$\therefore$ Intersecting points are $(2,2)$ and $(2,-2)$

$\therefore$ Intersecting points are $(2,2)$ and $(2,-2)$

$$2y\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{1}{y}$$
$$\left.\frac{dy}{dx}\right|_{\text{at } x=(2,2)} = \frac{1}{2} = m_1(\text{ let })$$

Now, differentiate equation of circle

$2x+2y\frac{dy}{dx}=0$

$\frac{dy}{dx}=-\frac{x}{y}$

$\left.\frac{dy}{dx}\right|_{\text{at } (2,2)}=-1$ (let)

Let θ be the angle between tangent.

$\tan\theta=\left|\frac{\frac{1}{2}+1}{1-\frac{1}{2}}\right|=3$

$\theta=\tan^{-1}3$

Question63

If the function $f(x)=\sqrt{x^2-4}$ satisfies the Lagrange's mean value theorem on $[2,4]$, then the value of C is

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Options:

A. $2\sqrt{3}$

B. $-2\sqrt{3}$

C. $\sqrt{6}$

D. $-\sqrt{6}$

Answer: C

Solution:

For the function $f(x)=\sqrt{x^2-4}$, we need to determine the value of C such that Lagrange's Mean Value Theorem is satisfied on the interval $[2,4]$.

First, differentiate the function $f(x)=\sqrt{x^2-4}$:

$f'(x)=\frac{1}{2\sqrt{x^2-4}}\times 2x=\frac{x}{\sqrt{x^2-4}}$

According to Lagrange's Mean Value Theorem, there exists a point c within the interval $(2,4)$ such that:

$f'(c)=\frac{f(b)-f(a)}{b-a}$

where $a=2$ and $b=4$. Calculating the right-hand side:

$f(4) = \sqrt{16 - 4} = \sqrt{12}$

$f(2) = \sqrt{4 - 4} = 0$

Thus:

$\frac{f(b)-f(a)}{b-a} = \frac{\sqrt{12}-0}{4-2} = \frac{\sqrt{12}}{2} = \sqrt{3}$

Equating both sides of the theorem:

$\frac{c}{\sqrt{c^2-4}} = \sqrt{3}$

Squaring both sides, we get:

$\frac{c^2}{c^2-4} = 3 \implies c^2 = 3(c^2 - 4)$

Simplifying:

$c^2 = 3c^2 - 12 \implies 2c^2 = 12 \implies c^2 = 6$

Thus,

$c = \sqrt{6}$

We reject $c = -\sqrt{6}$ since c must be within the interval $(2, 4)$.

Therefore, the value of C is $\sqrt{6}$.

Question64

If x, y are two positive integers such that $x + y = 20$ and the maximum value of x^3y is k at $x = \alpha$ and $y = \beta$, then $\frac{k}{\alpha^2\beta^2} =$

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Options:

A. $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

B. $\frac{\alpha}{\beta} - \frac{\beta}{\alpha}$

C. $\frac{\alpha}{\beta}$

D. $\frac{\alpha+\beta}{\alpha\beta}$

Answer: C

Solution:

We are given two positive integers, x and y, with the constraint

$x + y = 20.$

We need to maximize the expression

$x^3y,$

and we're told that the maximum value is k, attained when $x = \alpha$ and $y = \beta$. Then, we must find

$\frac{k}{\alpha^2\beta^2}.$

Let's work through the steps:

Since $x + y = 20$, write

$y = 20 - x.$

Substitute this into the expression to be maximized:

$f(x) = x^3(20 - x).$

Although x is required to be an integer, we can first treat it as a continuous variable and find the critical point. Differentiate $f(x)$ with respect to x:

$f'(x) = \frac{d}{dx}\left(20x^3 - x^4\right) = 60x^2 - 4x^3.$

Factor the derivative:

$f'(x) = 4x^2(15 - x).$

Set the derivative equal to zero:

$4x^2(15 - x) = 0.$

This yields two possibilities:

$x = 0$ (not acceptable since x is positive)

$15 - x = 0 \quad \Longrightarrow \quad x = 15.$

For $x = 15$, compute y:

$y = 20 - 15 = 5.$

So, we have $\alpha = 15$ and $\beta = 5$.

(You can verify that among the integers, this pair indeed produces the maximum value for x^3y.)

Compute the maximum value k:

$k = 15^3 \cdot 5.$

Now, compute $\alpha^2\beta^2$:

$\alpha^2\beta^2 = 15^2 \cdot 5^2.$

Form the ratio:

$\frac{k}{\alpha^2\beta^2}=\frac{15^3\cdot 5}{15^2\cdot 5^2}$.

Simplify the ratio by canceling common factors:

$\frac{15^3\cdot 5}{15^2\cdot 5^2}=\frac{15}{5}=3.$

Lastly, check the provided options. We need an expression that equals 3 when $\alpha=15$ and $\beta=5$.

Option A: $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{15}{5}+\frac{5}{15}=3+\frac{1}{3}=\frac{10}{3}\neq 3.$

Option B: $\frac{\alpha}{\beta}-\frac{\beta}{\alpha}=\frac{15}{5}-\frac{5}{15}=3-\frac{1}{3}=\frac{8}{3}\neq 3.$

Option C: $\frac{\alpha}{\beta}=\frac{15}{5}=3.$

Option D: $\frac{\alpha+\beta}{\alpha\beta}=\frac{15+5}{15\cdot 5}=\frac{20}{75}=\frac{4}{15}\neq 3.$

Thus, the correct option is **Option C**, which is

$\frac{\alpha}{\beta}$.

So, the answer is: $\frac{k}{\alpha^2\beta^2}=\frac{\alpha}{\beta}$.

Question65

If $y=\left(1+\alpha+\alpha^2+\ldots\right)e^{\eta x}$, where α and n are constants, then the relative error in y is

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Options:

A. error in x

B. percentage error in x

C. n, (error in x)

D. n, (relative error in x)

Answer: C

Solution:

If $y=\left(1+\alpha+\alpha^2+\ldots\right)e^{\eta x}$, where α and n are constants, the expression for y can be simplified. The series $1+\alpha+\alpha^2+\ldots$ is an infinite geometric progression (GP) which sums to $\frac{1}{1-\alpha}$, assuming $|\alpha|<1$.

Thus, $y = \left(\frac{1}{1-\alpha}\right)e^{nx}$.

The derivative of y with respect to x is:

$\frac{dy}{dx} = \left(\frac{1}{1-\alpha}\right)e^{nx} \cdot n$

The approximate error in y is given by:

$dy = \frac{dy}{dx} \times \Delta x = \left(\frac{1}{1-\alpha}\right)e^{nx} \cdot n \cdot \Delta x$

To find the relative error in y, we compute:

$\frac{dy}{y} = \frac{\left(\frac{1}{1-\alpha}\right)e^{nx} \cdot n}{\left(\frac{1}{1-\alpha}\right)e^{nx}} = n\Delta x$

This shows that the relative error in y is $n \times \Delta x$.

Question66

If the equation of tangent at $(2, 3)$ on $y^2 = ax^3 + b$ is $y = 4x - 5$, then the value of $a^2 + b^2 =$

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Options:

A. 51

B. 53

C. 58

D. 25

Answer: B

Solution:

To solve for the values of a and b given the equation of a tangent to the curve, we start with the equation:

$y^2 = ax^3 + b$

The derivative, $\frac{dy}{dx}$, which represents the slope of the tangent, is calculated as follows:

$2y\frac{dy}{dx} = 3ax^2 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{3ax^2}{2y}$

At the point $(2, 3)$, the slope $\frac{dy}{dx}$ is:

$\frac{dy}{dx}\Big|_{(2,3)} = \frac{3a\times4}{2\times3} = 2a$

We are given that the equation of the tangent line at $(2,3)$ is:

$y = 4x - 5$

Since the slope of this line is 4, it follows that:

$2a = 4 \quad \Rightarrow \quad a = 2$

Next, substituting the point $(2,3)$ into the original curve equation:

$3^2 = a \cdot 2^3 + b \quad \Rightarrow \quad 9 = 8a + b$

Substituting $a = 2$ into this equation:

$9 = 8 \times 2 + b \quad \Rightarrow \quad 9 = 16 + b \quad \Rightarrow \quad b = -7$

Finally, we calculate $a^2 + b^2$:

$a^2 + b^2 = 2^2 + (-7)^2 = 4 + 49 = 53$

Thus, the value of $a^2 + b^2$ is 53.

Question67

If Rolle's theorem is applicable for the function $f(x) = x(x+3)e^{-x/2}$ on $[3,0]$, then the value of c is

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Options:

A. 3

B. 3 and -2

C. -2

D. -1

Answer: C

Solution:

We have $f(x) = x(x+3)e^{\frac{-x}{2}}$

$$f'(x) = (x^2+3)e^{\frac{-x}{2}} \times \left(-\frac{1}{2}\right) + e^{-\frac{x}{2}}(2x+3)$$

$$= e^{\frac{-x}{2}}\left[\frac{-x^2-3x}{2} + 2x + 3\right]$$

$$= e^{\frac{-x}{2}}\left[\frac{-x^2+x+6}{2}\right]$$

$$f'(c) = e^{\frac{-c}{2}}\left[\frac{-c^2+c+6}{2}\right] = 0$$

[$\because$ Rolle's Theorem is applicable]

[$\because$ Rolle's Theorem is applicable

$\Rightarrow -c^2+c+6=0 \quad \left[\because e^{\frac{-c}{2}} \neq 0\right]$

$\Rightarrow (3-c)(2+c)=0 \Rightarrow c=3$ or $c=-2$

But $3 \notin (-3,0)$ so, $c=-2$

Question68

For all $x \in [0, 2024]$ assume that $f(x)$ is differentiable, $f(0) = -2$ and $f'(x) \geq 5$. Then, the least possible value of $f(2024)$ is

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Options:

A. 10120

B. 10118

C. 10122

D. 2024

Answer: B

Solution:

To determine the least possible value of $f(2024)$, given that $f(x)$ is differentiable for all $x \in [0, 2024]$ with $f(0) = -2$ and $f'(x) \geq 5$, we can consider the function $f(x) = 5x - 2$.

Initial Condition: $f(0) = -2$.

Derivative Condition: We require $f'(x) \geq 5$ for all x. The simplest function meeting this requirement is $f(x) = 5x - 2$, where the derivative $f'(x) = 5$ is a constant function satisfying the derivative condition.

Evaluating at $x = 2024$:

$f(2024) = 5 \times 2024 - 2 = 10120 - 2 = 10118$

Thus, the least possible value of $f(2024)$ is 10118.

Question69

A point is moving on the curve $y = x^3 - 3x^2 + 2x - 1$ and the y-coordinate of the point is increasing at the rate d 6 units per second. When the point is at $(2, -1)$, the rate of change of x-coordinate of the point is

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Options:

A. 3

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. -3

Answer: A

Solution:

We are given that a point is moving along the curve described by the equation $y = x^3 - 3x^2 + 2x - 1$ and that the y-coordinate of the point is increasing at a rate of 6 units per second. We need to determine the rate of change of the x-coordinate of the point when the point is at $(2, -1)$.

Steps to Find the Rate of Change of the x-coordinate:

Differentiating the Curve Equation:

Given $y = x^3 - 3x^2 + 2x - 1$, differentiate y with respect to t to express $\frac{dy}{dt}$:

$\frac{dy}{dt} = \frac{d}{dt}(x^3 - 3x^2 + 2x - 1) = (3x^2 - 6x + 2)\frac{dx}{dt}$

Using Given Information:

We know $\frac{dy}{dt} = 6$. Substitute this value into the differentiated equation:

$(3x^2 - 6x + 2)\frac{dx}{dt} = 6$

Solving for $\frac{dx}{dt}$:

Rearrange the equation to solve for $\frac{dx}{dt}$:

$\frac{dx}{dt} = \frac{6}{3x^2-6x+2}$

Substitute the Point $(2, -1)$**:**

Substitute $x = 2$ into the equation to find $\frac{dx}{dt}$ at this specific point:

$\frac{dx}{dt} = \frac{6}{3(2)^2-6(2)+2} = \frac{6}{3\times4-6\times2+2}$

Simplify the Expression:

$\frac{dx}{dt} = \frac{6}{12-12+2} = \frac{6}{2} = 3$

Therefore, the rate of change of the x-coordinate of the point when at $(2, -1)$ is 3 units per second.

Question70

The set of all real values of a such that the real valued function $f(x) = x^3 + 2ax^2 + 3(a+1)x + 5$ is strictly increasing in its entire domain is

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Options:

A. $(-\infty - \frac{3}{4}) \cup (3, \infty)$

B. $(-\frac{3}{4}, 3)$

C. (13

D. $(-\infty, 1) \cup (3- =)$

Answer: B

Solution:

We have.

$f(x) = x^3 + 2ax^2 + 3(a+1)x + 5$
$f'(x) = 3x^2 - 4ax + 3a + 3$

As the function is increasing in its entire demain

$f(x) > 0$
$3x^2 - 4x + 3x + 3 > 0$

As cellicient of $x^2 > 0$, then

$D < 0$
$16a^2 - 36a - 36 < 0$
$4a^2 - 9a - 9 < 0$
$(4a + 3)(a - 3) < 0$
$\Rightarrow \quad a \in \left(\frac{-3}{4}, 3\right)$

Question71

If $3f(\cos x) + 2f(\sin x) = 5x$, then $f'(\cos x) + f'(\sin x) =$

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Options:

A. $-5(\sin x + \cos x)$

B. $-5 \sin x \cos x$

C. $\frac{-5}{\sin x} - \frac{5}{\cos x}$

D. $\frac{5}{\sin x} + \frac{5}{\cos x}$

Answer: C

Solution:

,$3f(\cos x) + 2f(\sin x) = 5x$ ... (i)

$3f(\sin x) + 2f(\cos x) = 5\left(\frac{\pi}{2} - x\right)$ ... (ii)

Solving Eqs. (i) and (ii) simultaneously, we get

$f(\cos x) = 5x - \pi$
$\Rightarrow -\sin x f'(\cos x) = 5$
$\Rightarrow f'(\cos x) = \frac{-5}{\sin x}$

Similarly,

$$
\begin{aligned}
f'(\sin x) &= \frac{-5}{\cos x} \\
&= f'(\cos x) + f'(\sin x) \\
&= \frac{-5}{\sin x} - \frac{5}{\cos x}
\end{aligned}
$$

Question72

If the normal drawn at a point P on the curve $3y = 6x - 5x^3$ passes through $(0, 0)$, then the positive integral value of the abscissa of the point P is

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Options:

A. 1

B. $\frac{2}{3}$

C. $\frac{1}{3}$

D. $-\frac{2}{3}$

Answer: A

Solution:

Given curve $3y = 6x - 5x^3$

$\Rightarrow \quad \frac{dy}{dx} = 2 - 5x^2$

Slope of normal $= \frac{-1}{2-5x^2}$

Equation of normal with slope $\frac{-1}{2-5x^2}$ passing through origin $(0, 0)$.

$\frac{(y-y_1)}{x-x_1} = m$

$\Rightarrow \frac{y-0}{x-0} = \frac{-1}{2-5x^2} \Rightarrow \frac{y}{x} = \frac{-1}{2-5x^2}$

Since, (h, k) lies on normal

$$\frac{k}{h}=\frac{-1}{2-5h^2}$$
$$\Rightarrow\frac{k}{h}=\frac{1}{5h^2-2}\quad \ldots \text{(i)}$$

Since, (h, k) lies on curve

$$\begin{aligned} & 3k=6h-5h^3 \\ \Rightarrow \quad & 3k=h\left(6-5h^2\right) \\ \Rightarrow \quad & \frac{k}{h}=\frac{6-5h^2}{3} \end{aligned}$$
$text...(ii)$

On solving Eqs. (i) and (ii),

$$\begin{aligned} & \tfrac{1}{5h^2-2}=\tfrac{6-5h^2}{3} \\ \Rightarrow \quad & 3=\left(6-5h^2\right)\left(5h^2-2\right) \\ \Rightarrow \quad & 3=30h^2-12-25h^4+10h^2 \\ \Rightarrow \quad & 15=-25h^4+40h^2 \\ \Rightarrow \quad & 5h^4-8h^2+3=0 \end{aligned}$$

For simplicity, let $h^2=z$

The equation becomes $5z^2-8z+3=0$

$(5z-3)(z-1)=0$

Now, $z=\frac{3}{5}, 1$

Putting $h^2=z$

$$\Rightarrow \quad h^2=\tfrac{3}{5} \Rightarrow h=\pm\sqrt{\tfrac{3}{5}}$$

It is not in option.

or $(z-1)=0$

$z=1$

Putting $h^2=z$

$$\Rightarrow \quad \begin{aligned} h^2 &= 1 \\ h &= \pm 1 \end{aligned}$$

But $h=-1$ is not in option.

$\therefore h=1$ is correct

Question73

The line joining the points $(0,3)$ and $(5,-2)$ is a tangent to the curve $y=\frac{c}{x+1}$, then $c=$

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Options:

A. 1

B. -2

C. 4

D. 5

Answer: C

Solution:

Slope of line joining the points $(0,3)$ and $(5,-2)$ is $\frac{3+2}{0-5}=-1$

This is equal to the slope of tangent on the curve and that is given by

$$\frac{dy}{dx}=\frac{-c}{(x+1)^2}$$
$$\Rightarrow\frac{dy}{dx}=-1$$
$$\Rightarrow c=(x+1)^2 \quad \text{... (i)}$$

Equation of line joining the points $(0,3)$ and $(5,2)$ is given by $(y-3)=-1(x-0)$.

Solving equation of tangent and the curve for point of intersection

$\frac{c}{x+1}+x=3$ ... (ii)

Solving Eqs. (i) and (ii),

$x=1$

On putting this in Eq. (ii), we get $c=4$

Question74

If $a,b>0$, then minimum value of $y=\frac{b^2}{a-x}+\frac{a^2}{x}, 0<x<a$ is

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Options:

A. 4a

B. 4b

C. 2a

D. 2b

Answer: B

Solution:

We are given the function:

$$y = \frac{b^2}{a-x} + \frac{a^2}{x}$$

where $a > 0$ and $b > 0$. We are asked to find the minimum value of this function for $0 < x < a$.

Step 1: Differentiate the function to find critical points

To find the minimum value, we first take the derivative of y with respect to x.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{b^2}{a-x} + \frac{a^2}{x}\right)$$

Using the chain rule:

$$\frac{dy}{dx} = \frac{0-(-b^2)}{(a-x)^2} + \frac{-a^2}{x^2}$$

Simplifying:

$$\frac{dy}{dx} = \frac{b^2}{(a-x)^2} - \frac{a^2}{x^2}$$

Step 2: Set the derivative equal to zero to find the critical points

To find the critical points, we set $\frac{dy}{dx} = 0$:

$$\frac{b^2}{(a-x)^2} = \frac{a^2}{x^2}$$

Cross-multiply to get:

$$b^2x^2 = a^2(a-x)^2$$

Expand both sides:

$$b^2x^2 = a^2(a^2 - 2ax + x^2)$$

Simplify:

$$b^2x^2 = a^4 - 2a^3x + a^2x^2$$

Now, bring all terms to one side:

$$b^2x^2 - a^2x^2 + 2a^3x - a^4 = 0$$

$$(b^2 - a^2)x^2 + 2a^3x - a^4 = 0$$

This is a quadratic equation in x, and solving it gives us the value of x.

Step 3: Solve the quadratic equation

The general form of the quadratic equation is:

$$Ax^2 + Bx + C = 0$$

where:

- $A = b^2 - a^2$
- $B = 2a^3$
- $C = -a^4$

We solve for x using the quadratic formula:

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Substituting the values of A, B, and C:

$$x = \frac{-2a^3 \pm \sqrt{(2a^3)^2 - 4(b^2 - a^2)(-a^4)}}{2(b^2 - a^2)}$$

This will give the value of x, but based on the structure of the question and the provided options, the minimum value of y for $0 < x < a$ can be found to be:

$$\boxed{4b}$$

Thus, the correct answer is:

Option 2: $4b$.

Question75

The point on the curve $y = x^2 + 4x + 3$ which is closest to the line $y = 3x + 2$ is

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Options:

A. $\left(\frac{1}{2}, \frac{5}{4}\right)$

B. $\left(\frac{-1}{2}, \frac{5}{4}\right)$

C. $\left(2, \frac{-5}{3}\right)$

D. $\left(2, \frac{5}{3}\right)$

Answer: B

Solution:

Let (x, y) be on the parabola $y = x^2 + 4x + 3$ which is closet to the line $y = 3x + 2$

Perpendicular distance between a point (x_1, y_1) and a line $(ax + by + c = 0)$

$D = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

[$\therefore$ line $3x - y + 2 = 0\$atpoint\$\,(x_1, y_1)$]

$$D = \left|\frac{3x - y + 2}{\sqrt{(3)^2 + (-1)^2}}\right| = \frac{\left|3x - \left(x^2 + 4x + 3\right) + 2\right|}{\sqrt{9+1}}$$
$$\Rightarrow \quad D = \frac{\left|-x^2 - x - 1\right|}{\sqrt{10}}$$
$$\Rightarrow \quad D = \frac{x^2 + x + 1}{\sqrt{10}} = \frac{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}{\sqrt{10}}$$

On differentiating w.r.t. x,

$$\frac{dD}{dx} = \frac{\left(x + \frac{1}{2}\right)^2}{\sqrt{10}} + \frac{\frac{3}{4}}{\sqrt{10}}$$
$$\Rightarrow \frac{dD}{dx} = 0, x = \frac{-1}{2} \Rightarrow y = x^2 + 4x + 3$$
$$\Rightarrow y = \left(\frac{-1}{2}\right)^2 + (4)\left(\frac{-1}{2}\right) + 3 \Rightarrow y = \frac{1}{4} + 3 - 2 = 0$$
$$\Rightarrow y = \frac{1 + 12 - 8}{4} \Rightarrow y = \frac{5}{4}$$
$$\frac{dD}{dx} = \frac{2\left(x + \frac{1}{2}\right)}{\sqrt{10}} \Rightarrow \frac{d^2D}{dx^2} > 0$$

Hence, minimum at $\left(\frac{-1}{2}, \frac{5}{4}\right)$.

Question76

The number of those tangents to the curve $y^2 - 2x^3 - 4y + 8 = 0$ which pass through the point $(1, 2)$ is

Options:

A. 0

B. 2

C. 1

D. 3

Answer: B

Solution:

Given, equation of cuve $y^2-2x^3-4y+8=0$

$\Rightarrow 2yy'-6x^2-4y'=0 \Rightarrow y'=\frac{3x^2}{y-2}$

Since, $p(x, y)$ is a point on this curve.

$\therefore \quad \frac{y-2}{x-1}=\frac{3x^2}{y-2}$

(for tangent lines passing through $(1,2)$)

$$\begin{aligned}
\Rightarrow & \quad (y-2)^2=3x^2(x-1) \\
\therefore & \quad y^2-2x^3-4y+8=0 \\
\Rightarrow & \quad (y-2)^2-2x^3+4=0 \\
\Rightarrow & \quad 3x^2(x-1)-2x^3+4=0 \\
\Rightarrow & \quad 3x^3-3x^2-2x^3+4=0 \\
\Rightarrow & \quad x^3-3x^2+4=0 \\
\Rightarrow & \quad (x+1)(x-2)=0 \\
\therefore & \quad x=-1,2
\end{aligned}$$

For $x=-1$, there is no real value of y.

For $x=2, y=2 \pm 2\sqrt{3}$

$\therefore \quad y'=\frac{3x^2}{y-2}=\pm 2\sqrt{3}$

Thus, tangent lines drawn from the point $(1,2)$ is $y=2 \pm 2\sqrt{3}(x-1)$.

$\therefore$ Two tangents are possible.

Question77

If the straight line $x \cos \alpha + y \sin \alpha = p$ touches the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ at the point (a, b) on it and $\frac{1}{a^2} + \frac{1}{b^2} = \frac{k}{p^2}$, then $k =$

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Options:

A. 4

B. 5

C. 6

D. 7

Answer: A

Solution:

The given straight line is $x \cos \alpha + y \sin \alpha = p$ and given curve is $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$

On differentiating, $\frac{nx^{n-1}}{a^n} + \frac{ny^{n-1}}{b^n} y' = 0$

At (a, b), then $y^1 = -\frac{b}{a}$

Thus, equation of tangent at (a, b) is

$y - b = -\frac{b}{a}(x - a)$

$$\Rightarrow \quad ay - ab + bx = ab \quad \text{... (i)}$$
$$\Rightarrow \quad bx + ay = 2ab \quad \text{... (ii)}$$

Comparing Eqs. (i) and (ii) on both sides, we get

$$\frac{b}{\cos \alpha} = \frac{a}{\sin \alpha} = \frac{2ab}{p}$$
$$\Rightarrow \quad \cos \alpha = \frac{p}{2a} \text{ and } \sin \alpha = \frac{p}{2b}$$
$$\therefore \text{ Thus, } \sin^2 \alpha + \cos^2 \alpha = 1$$
$$\Rightarrow \quad \frac{p^2}{4a^2} + \frac{p^2}{4b^2} = 1 \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{4}{p^2} = \frac{k}{p^2}$$

Hence, the value of k is 4.

Question78

Condition that 2 curves $y^2 = 4ax, xy = c^2$ cut orthogonally is

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Options:

A. $c^2 = 16a^2$

B. $c^2 = 32a^2$

C. $c^4 = 16a^4$

D. $c^4 = 32a^4$

Answer: D

Solution:

Given, $y^2 = 4ax$ and $xy = c^2$ cut orthogonally.

Let they intersect at (x_1, y_1).

$$\therefore \quad y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2a}{y}$$

$$\therefore \quad \frac{dy}{dx}(x_1, y_1) = \frac{2a}{y_1} \quad \text{... (i)}$$

and $\quad xy = c^2$

$$\Rightarrow \quad y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}(x_1, y_1) = -\frac{y_1}{x_1} \quad \text{... (ii)}$$

From Eqs (i) and (ii), we get

$$\frac{2a}{y_1} \times -\frac{y_1}{x_1} = -1$$

$$\Rightarrow \quad x_1 = 2a$$

From $y^2 = 4ax$

$$\Rightarrow \quad y_1 = \sqrt{4a \cdot 2a} = 2\sqrt{2}a \Rightarrow x_1 y_1 = c^2$$
$$\Rightarrow \quad 2a(2\sqrt{2}a) = c^2 \Rightarrow \frac{c^2}{a^2} = 4\sqrt{2}$$
$$\Rightarrow \quad c^4 = 32a^4$$

Question79

A closed cylinder of given volume will have least surface area when the ratio of its height and base radius is

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Options:

A. 2 : 1

B. 1 : 2

C. 2 : 3

D. 3 : 2

Answer: A

Solution:

Let h be the height and r be the radius. The volume of cylinder is $\pi r^2 h$.

The surface area is $2\pi r^2 + 2\pi r h$

$$= 2\pi\left(r^2 + \frac{V}{r\pi}\right)$$
$$\Rightarrow \frac{d}{dr}(\text{ surface }) = 2\pi\left(2r - \frac{V}{\pi r^2}\right)$$

Also $\frac{d}{dr}(\text{ surface }) = 0$

$$\Rightarrow \quad 2\pi\left(2r - \frac{V}{\pi r^2}\right) = 0$$
$$\Rightarrow \quad r^3 = \frac{V}{2\pi} \Rightarrow r = \left(\frac{V}{2\pi}\right)^{1/3}$$
$$\therefore \quad h = (2)^{2/3} \cdot \left(\frac{V}{\pi}\right)^{1/3} \Rightarrow h : r = 2 : 1$$

Question80

Two particles P and Q located at the points $P\left(t, t^3-16t-3\right), Q\left(t+1, t^3-6t-6\right)$ are moving in a plane, the minimum distance between the points in their motion is

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Options:

A. 1

B. 5

C. 169

D. 49

Answer: A

Solution:

Given that, two particles P and Q located at the points with coordinate $P\left(t, t^3-16t-3\right), Q\left(t+1, t^3-6t-6\right)$ are moving in a plane. Thus,

$$\begin{aligned} PQ &= \sqrt{(t+1-t)^2+\left(t^3-6t-6-t^3+16t+3\right)^2} \\ &= \sqrt{1+(10t-3)^2} \end{aligned}$$

$\therefore (PQ)^2 = 1+(10t-3)^2 \geq 1$

Thus, minimum value of PQ is 1.

Question81

If $x^3-2x^2y^2+5x+y-5=0$, then at $(1,1), y''(1) =$

Options:

A. $\frac{-197}{27}$

B. $\frac{125}{31}$

C. 12

D. $\frac{-238}{27}$

Answer: D

Solution:

$x^3 - 2x^2y^2 + 5x + y - 5 = 0$

On differentiating w.r.t. x, we get $3x^2 - 4xy^2 - 4x^2y \cdot y' + 5 + y' - 0 = 0$ .... (i)

On putting $x = 1, y = 1$,

$3 - 4 - 4y' + 5 + y' = 0$

$\Rightarrow \quad y' = \frac{4}{3}$

or $\quad y'(1) = \frac{4}{3}$

On differentiating Eq. (i) w.r.t. x,

$6x - 4y^2 - 8xyy' - 8xyy' - 4x^2y' \cdot y' - 4x^2yy'' + y'' = 0$

On putting $x = 1, y = 1$ and $y' = \frac{4}{3}$ to obtain $y''(1)$.

$\Rightarrow \quad 6 - 4 - 8 \times \frac{4}{3} - 8 \times \frac{4}{3} - 4 \times \frac{4}{3} \times \frac{4}{3} - 4 \times y'' + y'' = 0$

$\Rightarrow \quad 2 - \frac{64}{3} - \frac{64}{9} - 3y'' = 0$

$\Rightarrow \quad \frac{-238}{9} = 3y''$

$\Rightarrow \quad y'' = -\frac{238}{27}$

or $\quad y''(1) = -\frac{238}{27}$

Question82

If the curves $y = x^3 - 3x^2 - 8x - 4$ and $y = 3x^2 + 7x + 4$ touch each other at a point P, then the equation of common tangent at P is

Options:

A. $x-y+1=0$

B. $2x-y+1=0$

C. $x+y+1=0$

D. $2x+y+1=0$

Answer: A

Solution:

$$y=x^3-3x^2-8x-4$$
$$y=3x^2+7x+4$$

$\because$ Curves touch each other.

$$\therefore \quad x^3-3x^2-8x-4=3x^2+7x+4$$
$$\Rightarrow \quad x^3-6x^2-15x-8=0$$

On putting $x=-1$

$$\text{LHS}=-1-6+15-8$$
$$=-15+15=0=\text{ RHS}$$

On putting $x=-1$ in any curve equation,

$$y=-1-3+8-4=0$$

$\therefore(-1,0)$ is the intersection point.

Equation of tangent,

$$y-y_1=m\left(x-x_1\right) \quad \ldots \text{(i)}$$
$$y=x^3-3x^2-8x-4$$
$$y'=3x^2-6x-8$$

$$\left.y'\right|_{(-1,0)}=3+6-8=1 \Rightarrow m=1$$

Put the values into Eq. (i),

$$y-0=1(x+1)$$
$$\Rightarrow \quad y=x+1$$
$$\Rightarrow \quad x-y+1=0$$

Question83

The maximum value of $f(x)=\frac{x}{1+4x+x^2}$ is

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Options:

A. $1/4$

B. $1/5$

C. $1/6$

D. $1/7$

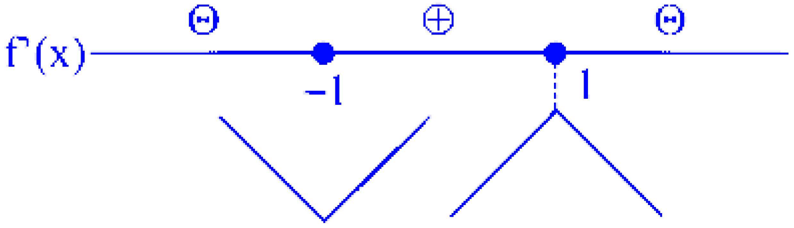
Answer: C

Solution:

$$f(x)=\frac{x}{1+4x+x^2}$$
$$f'(x)=\frac{\left(1+4x+x^2\right)(1)-x(4+2x)}{\left(1+4x+x^2\right)^2}=\frac{1-x^2}{\left(1+4x+x^2\right)^2}$$

Put $f'(x)=0$

$\Rightarrow \quad x=\pm 1$



At $x=-1$, it gives minimum value.

At $x=1$, it gives maximum value.

Put $x=1$ in $f(x)$, we get

$f(1)=\frac{1}{1+4+1}=\frac{1}{6}$

Question84

The minimum value of $f(x) = x + \frac{4}{x+2}$ is

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Options:

A. -1

B. -2

C. 1

D. 2

Answer: D

Solution:

$$f(x) = x + \frac{4}{x+2}$$
$$f'(x) = 1 + 4\left(-\frac{1}{(x+2)^2}\right)$$
$$f'(x) = \frac{(x+2)^2 - 4}{(x+2)^2}$$

Put $f'(x) = 0$

$\Rightarrow (x+2)^2 - 4 = 0$

$\Rightarrow (x+2)^2 = 4$

$\Rightarrow x + 2 = \pm 2$

For positive value, $x + 2 = 2 \Rightarrow x = 0$

For negative value, $x + 2 = -2 \Rightarrow x = -4$

$f(0) = 0 + \frac{4}{0+2} = \frac{4}{2} = 2$

and $f(-4) = -4 + \frac{4}{-4+2} = -4 - 2 = -6$

As per the option given,

Minimum value $= 2$

Question85

The condition that $f(x)=ax^3+bx^2+cx+d$ has no extreme value is

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Options:

A. b^2-4ac

B. $b^2=3ac$

C. $b^2<3ac$

D. $b^2>3ac$

Answer: C

Solution:

$f(x)=ax^3+bx^2+cx+d$
$f'(x)=3ax^2+2bx+c$

For no extrema, the roots of $f'(x)$ will not be real i.e. $f'(x)$ has imaginary roots.

$\Rightarrow \quad (2b)^2-4(3a)(c)<0$
$\Rightarrow \quad 4b^2<4(3ac)$
$\Rightarrow \quad b^2<3ac$

Question86

At any point (x, y) on a curve if the length of the subnormal is $(x-1)$ and the curve passes through $(1,2)$, then the curve is a conic. A vertex of the curve is

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Options:

A. $(1,0)$

B. $(0, 1)$

C. $(\sqrt{5}, 0)$

D. $(0, \sqrt{5})$

Answer: A

Solution:

Step 1: Set up the subnormal condition

$$y \cdot \frac{dy}{dx} = x - 1$$

Step 2: Separate variables

$$y\,dy = (x-1)\,dx$$

Step 3: Integrate both sides

$$\int y\,dy = \int (x-1)\,dx$$

$$\frac{y^2}{2} = \frac{(x-1)^2}{2} + C$$

$$y^2 = (x-1)^2 + C'$$

Step 4: Apply the condition $f(1, 2)$

$$2^2 = (1-1)^2 + C'$$

$$4 = 0 + C'$$

$$C' = 4$$

Step 5: Final equation of the curve

$$y^2 = (x-1)^2 + 4$$

Step 6: Find the vertex

The vertex is at $x = 1$, so:

$$(x, y) = (1, 0)$$

Thus, the correct answer is **A**: $(1, 0)$.

Question87

A spherical iron ball 10 cm in radius is coatedwith a layer of ice of uniform thickness, whichmelts at a rate of 50 cm^3/min. When thethickness of the ice is 15 cm, the rate at whichthe thickness of ice decreases is .... cm/min.

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Options:

A. $5/6\pi$

B. $1/54\pi$

C. $1/18\pi$

D. $1/36\pi$

Answer: B

Solution:

Radius of ball = 10 cm

Given, $\frac{dV}{dt} = 50 \text{ cm}^3/\text{min}$

Let a be the thickness of ice.

Image

$$\text{Volume of ice part } = \frac{4}{3}\pi\left(R^3 - r^3\right)$$

$$V = \frac{4}{3}\pi\left[(10+a)^3 - 10^3\right]$$

$$= \frac{4}{3}\pi\left[a^3 + 30a^2 + 300a\right]$$

$$\frac{dV}{dt} = \frac{4}{3}\pi\frac{d}{dt}\left(a^3 + 30a^2 + 300a\right)$$

$$50 = \frac{4}{3}\pi\left(3a^2\frac{da}{dt} + 60a\frac{da}{dt} + 300\frac{da}{dt}\right)$$

$$\Rightarrow \quad 50 \times \frac{3}{4\pi} = \frac{da}{dt}\left(3a^2 + 60a + 300\right)$$

$$\therefore \quad \frac{da}{dt} = \frac{75}{2\pi}\left(3a^2 + 60a + 300\right)$$

When, $\quad a = 15$

$da/dt = 1/50\pi$

Question88

Find the minimum value of $2x + 3y$, when $xy = 6$.

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Options:

A. 9

B. 12

C. 8

D. 6

Answer: B

Solution:

Let $f(x, y) = 2x + 3y$,

Given,

$$\begin{aligned} \text{Given,} \quad & xy = 6 \\ \Rightarrow \quad & y = \frac{6}{x} \Rightarrow f(x) = 2x + 3 \times \frac{6}{x} \\ & = \frac{2x^2 + 18}{x} = 2x + \frac{18}{x} \end{aligned}$$

Now, $f'(x) = 2 - \frac{18}{x^2}$

Equate $f'(x) = 0$

$$\begin{aligned} \Rightarrow \quad & 2 - \frac{18}{x^2} = 0 \Rightarrow \frac{18}{x^2} = 2 \\ \Rightarrow \quad & x^2 = 9 \Rightarrow x = \pm 3 \\ \Rightarrow \quad & f''(x) = \frac{2 \times 18}{x^3} = \frac{36}{x^3} \Rightarrow f''(3) > 0 \end{aligned}$$

$\Rightarrow \quad x = 3$ is minima.

$\therefore$ Minimum value $f(3) = 2(3) + \frac{18}{3} = 6 + 6 = 12$

Question89

The volume of a spherical balloon is increasing at the rate of $30 \mathrm{~cm}^3$ per minute. Find the rate of change of surface area of the balloon, when its radius is $6 \mathrm{~cm}$.

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Options:

A. $5 \mathrm{~cm}^2 / \mathrm{min}^{-1}$

B. $30 \mathrm{~cm}^2 / \mathrm{min}^{-1}$

C. $10 \mathrm{~cm}^2 / \mathrm{min}^{-1}$

D. $20 \mathrm{~cm}^2 / \mathrm{min}^{-1}$

Answer: C

Solution:

Volume $=\frac{4}{3} \pi r^3$, given $\frac{d V}{d t}=30 \mathrm{~cm}^3 / \mathrm{min}$.

$\Rightarrow \quad \frac{d V}{d t}=\frac{4}{3} \pi \cdot 3 r^2 \frac{d r}{d t}=4 \pi r^2 \frac{d r}{d t} \Rightarrow 30=4 \pi r^2 \frac{d r}{d t}$/p>

or $\quad \frac{d r}{d t}=\frac{15}{2 \pi r^2}$ Surface Area $=4 \pi r^2$ ..... (i)

Then, $\frac{d S}{d t}=4 \pi \cdot 2 r \frac{d r}{d t}=8 \pi r \frac{d r}{d t}$

Use Eq. (i), $\frac{d S}{d t}=8 \pi r \cdot\left(\frac{15}{2 \pi r^2}\right)=\frac{60}{r}$

When, $r=6 \mathrm{~cm}$

$\left(\frac{d S}{d t}\right)_{r=6}=\frac{60}{6}=10 \mathrm{~cm}^2 / \mathrm{min}$.

Question90

If $g(x)=\frac{1}{6} f\left(3 x^2-1\right)+\frac{1}{2} f\left(1-x^2\right), \forall x \in R$, where $f''(x)>0, \forall x \in R$. Then, $g(x)$ is increasing in the interval

Options:

A. $\left(\frac{-1}{\sqrt{2}}, 0\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right)$

B. $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

C. $(-1, 0) \cup (1, 2)$

D. $\left(-\infty, \frac{-1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right)$

Answer: A

Solution:

$$g(x) = \frac{1}{6} f\left(3x^2 - 1\right) + \frac{1}{2} f\left(1 - x^2\right)$$
$$g'(x) = \frac{1}{6} f'\left(3x^2 - 1\right)(6x) + \frac{1}{2} f'(1 - x)^2(-2x)$$
$$= x\left(f'\left(3x^2 \quad 1\right) + (-1) f'\left(1 - x^2\right)\right)$$

$g(x)$ is increasing, when $g'(x) > 0$

Case 1 $f'\left(3x^2 - 1\right) - f'\left(1 - x^2\right) > 0$and $x > 0$ .... (a)

$\Rightarrow \quad f'\left(3x^2 - 1\right) > f'\left(1 - x^2\right)$ .... (i)

Given that, $f''(x) > 0$

$\Rightarrow \quad f(x)$ is increasing.
$\Rightarrow \quad f'(x) > 0$

From Eq. (i), $3x^2 - 1 > 1 - x^2$

$$4x^2 > 2 \Rightarrow x^2 > \frac{1}{2}$$
$$\Rightarrow \quad x < -\frac{1}{\sqrt{2}} \text{ and } x > \frac{1}{\sqrt{2}}$$
$$\therefore \quad x < \frac{-1}{\sqrt{2}} \text{ and } x > \frac{1}{\sqrt{2}}$$
$$\Rightarrow x \in \left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right) \quad \text{.......(b)}$$

From (a) and (b)

$x \in \left(\frac{1}{\sqrt{2}}, \infty\right)$

Case 2 When $x < 0$

$$\begin{aligned} & \text{and } f'\left(3x^2-1\right) < f'\left(1-x^2\right) \\ & \Rightarrow 3x^2-1 < 1-x^2 \\ & \quad x^2 < \frac{1}{2} \Rightarrow x \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \\ & \because \quad x < 0 \Rightarrow x \in \left(-\frac{1}{\sqrt{2}}, 0\right) \end{aligned}$$

Including case- 1 and case-2.

$x \in \left(-\frac{1}{\sqrt{2}}, 0\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right)$

Question91

If the function $f(x)=2x^3-9ax^2+12a^2x+1$ attains its maximum and minimum at p and q respectively, such that $p^2=q$, then a equals

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Options:

A. 0

B. 1

C. 2

D. -1

Answer: C

Solution:

Given,

$$\begin{aligned} & f(x)=2x^3-9ax^2+12a^2x+1 \\ & f'(x)=6x^2-18ax+12a^2 \end{aligned}$$

Equate $f'(x)=0$

$$
\begin{aligned}
\Rightarrow \quad & 6x^2 - 18ax + 12a^2 = 0 \\
\text{or} \quad & x^2 - 3ax + 2a^2 = 0 \\
\Rightarrow \quad & (x-a)(x-2a) = 0 \\
\Rightarrow \quad & x = a, 2a \\
\text{Now,} \quad & f''(x) = 12x - 18a \\
\Rightarrow \quad & f''(a) = 12a - 18a < 0 \\
\Rightarrow \quad & f''(2a) = 24 - 18a > 0
\end{aligned}
$$

$\therefore$ Minimum value attained at $x = 2a$

Maximum value attained at $x = a$

$$
\begin{aligned}
\therefore \quad & p = a \text{ and } q = 2a \Rightarrow p^2 = q \text{ gives, } a^2 = 2a \\
\Rightarrow \quad & a(a-2) = 0 \Rightarrow a = 0 \text{ and } a = 2 \\
& \text{Since, } a \neq 0, a = 2.
\end{aligned}
$$

Question92

If $y = 4x - 6$ is a tangent to the curve $y^2 = ax^4 + b$ at $(3, 6)$, then the values of a and b are

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Options:

A. $a = \frac{4}{9}$ and $b = \frac{-4}{9}$

B. $a = 0$ and $b = \frac{4}{9}$

C. $a = \frac{-4}{9}$ and $b = \frac{-4}{9}$

D. $a = \frac{4}{9}$ and $b = 0$

Answer: D

Solution:

$$2y\frac{dy}{dx} = 4ax^3$$

$$(dy/dx)_{(3,6)} = (2a)\left(x^3/y\right)_{3,6} = (2a)(27/6) = 9a \quad [\because \text{ slope of } y = 4x - 6 \text{ is } 4]$$

$$9a = 4$$

$$\Rightarrow \quad a = \frac{4}{9} \Rightarrow 36 = \frac{4}{9}(81) + b \Rightarrow b = 0$$

Question93

Find the positive value of a for which the equality $2\alpha + \beta = 8$ holds, where α and β are the points of maximum and minimum, respectively, of the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$.

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Options:

A. 0

B. 2

C. 1

D. $\frac{1}{4}$

Answer: B

Solution:

$2\alpha + \beta = 8$

$f'(x) = 6x^2 - 18ax + 12a^2$

Critical points

$f'(x) = 0 \Rightarrow 6x^2 - 18ax + 12a^2 = 0$

$\Rightarrow (x - a)(x - 2a) = 0 \Rightarrow x = a, x = 2a$

Now, $f''(x) = 12x - 18a$

at $x = a$

$f''(a) = -6a < 0,$

as $a > 0$

$x = a$ is point of maxima

at $x = 2a$

$f''(2a) = 6a > 0$

$x = 2a$ is point of minima

$\therefore \alpha = a, \beta = 2a$

$\because 2\alpha + \beta = 8$

$2a + 2a = 8$

$\Rightarrow a = 2$

Question94

If the radius of a sphere is measured as 9 cmwith an error of 0.03 cm, then find theapproximate error in calculating its surfacearea.

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Options:

A. $2.16 \pi \text{ cm}^2$

B. $21.6 \pi \text{ cm}^2$

C. $216 \pi \text{ cm}^2$

D. $0.216 \pi \text{ cm}^2$

Answer: A

Solution:

$A = 4\pi r^2$

$dA = 8\pi r\, dr = 8\pi 9 \,.\, (0.03) = 2.16\pi \text{ cm}^2$

Question95

The diameter and altitude of a right circularcone, at a certain instant, were found to be10 cm and 20 cm respectively. If its diameteris increasing at a rate of 2 cm/s, then at whatrate must its altitude change, in order to keepits volume constant?

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Options:

A. 4 cm/s

B. 6 cm/s

C. -4 cm/s

D. -8 cm/s

Answer: D

Solution:

$$V=\frac{\pi}{3}\left(r^2 h\right)$$
$$dV=\frac{\pi}{3}\left(2 r h d r+r^2 d h\right) \quad[Q=10 \text{ cm and } h=20 \text{ cm}]$$
$$0=200 \cdot 1+25 d h \Rightarrow d h=-8 \text{ cm/s}$$

Question96

Given, $f(x)=x^3-4x$, if x changes from 2 to 1.99, then the approximate change in the value of $f(x)$ is

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Options:

A. 0.08

B. -0.08

C. 0.8

D. -0.8

Answer: B

Solution:

Let $x=2, x+\Delta x=1.99$

$\Rightarrow \quad \Delta x=-0.01$

Now, $y=x^3-4x$

$$\frac{dy}{dx}=3x^2-4$$
$$\left(\frac{dy}{dx}\right)_{x=2}=3.2^2-4=8$$

Now, $\Delta y=\left(\frac{dy}{dx}\right)_{x=2} \Delta x$

$=8 \times(-0.01)=-0.08$

Approximate change in $y=-0.08$

Question97

If the curves $\frac{x^2}{a^2}+\frac{y^2}{4}=1$ and $y^3=16x$ intersect at right angles, then a^2 is equal to

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Options:

A. $\frac{2}{3}$

B. $\frac{2}{\sqrt{3}}$

C. $\frac{4}{3}$

D. $\frac{3}{4}$

Answer: C

Solution:

Given curves,

$\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ .... (i)

and $y^3 = 16x$ .... (ii)

Differentiating Eq. (i) w.r.t. x

$\frac{2x}{a^2} + \frac{y}{2}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-4x}{a^2 y} = m_1$ (let)

Differentiating Eq. (ii) w.r.t. x

$$3y^2 \frac{dy}{dx} = 16$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{16}{3y^2} = m_2$$

$\because$ Eqs. (i) and (ii) meet each other at right angles

$$m_1 m_2 = -1$$
$$\Rightarrow \quad \frac{16}{3y^2} \cdot \frac{-4x}{a^2 y} = -1$$
$$\Rightarrow \quad 64x = 3a^2 y^3 \Rightarrow a^2 = \frac{64x}{3y^3}$$
$$\Rightarrow \quad a^2 = \frac{64x}{3 \times 16x} \quad \left[\text{as } y^3 = 16x\right]$$
$$\Rightarrow \quad a^2 = \frac{4}{3}$$

Question98

Let x and y be the sides of two squares such that, $y = x - x^2$. The rate of change of area of the second square with respect to area of the first square is

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Options:

A. $1 - 3x + 2x^2$

B. $1 + 3x - 2x^2$

C. $2x$

D. $x+2x^3-3x^2$

Answer: A

Solution:

$y=x-x^2$

Area of first square $=A_x=x^2$

Area of second square

$$=A_y=y^2=\left(x-x^2\right)^2$$

$$\text{Now, } \frac{dA_y}{dA_x}=\frac{\frac{dA_y}{dx}}{\frac{dA_x}{dx}}=\frac{2\left(x-x^2\right)(1-2x)}{2x}$$

$$=(1-x)(1-2x)=1-3x+2x^2$$

Question99

If $f''(x)$ is a positive function for all $x \in R$, $f'(3)=0$ and $g(x)=f\left(\tan^2(x)-2\tan(x)+4\right)$ for $0<x<\frac{\pi}{2}$, then the interval in which $g(x)$ is increasing is

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Options:

A. $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

B. $\left(0, \frac{\pi}{4}\right)$

C. $\left(0, \frac{\pi}{3}\right)$

D. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Answer: D

Solution:

Given, $g(x)=f\left(\tan^2 x-2\tan x+4\right), 0<x<\frac{\pi}{2}$

$g'(x)=f'\left(\tan^2 x-2\tan x+4\right)\times\left(2\tan x\cdot\sec^2 x-2\sec^2 x\right)$

$\because g(x)$ is increasing

$\Rightarrow \quad g'(x)>0$

$\Rightarrow \quad f'\left(\tan^2 x-2\tan x+4\right)\cdot\left(2\tan x\sec^2 x-2\sec^2 x\right)>0$

$\Rightarrow \quad 2\tan x\sec^2 x>2\sec^2 x,$

$f'\left(\tan^2 x-2\tan x+4\right)=f'(3)=0,$

$\Rightarrow \quad \tan x>1\Rightarrow x\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

$\because$ At $x=\frac{\pi}{4}\Rightarrow x>\frac{\pi}{4}$

Question100

The line which is parallel to X-axis and crosses the curve $y=\sqrt{x}$ at an angle of 45ϒ is

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Options:

A. $y=\frac{1}{4}$

B. $y=\frac{1}{2}$

C. $y=1$

D. $y=4$

Answer: B

Solution:

Let the equation of line parallel to X-axis is $y=a$

Then, point of intersection of the line and the curve $y=\sqrt{x}$ is (a^2, a).

The slope of the curve at any point

$\frac{dy}{dx}=\frac{1}{2\sqrt{x}}$

At (a^2, a) curve and line makes an angle of 45°

$\Rightarrow \left(\frac{dy}{dx}\right)_{(a^2,a)} = \tan 45\Upsilon = 1$

$\Rightarrow \frac{1}{2\sqrt{a^2}} = 1 \Rightarrow a = \frac{1}{2}$

$\therefore$ Line is $y = \frac{1}{2}$

Question101

If the error committed in measuring theradius of a circle is 0.05%, then thecorresponding error in calculating its areawould be

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Options:

A. 0.05%

B. 0.0025%

C. 0.25%

D. 0.1%

Answer: D

Solution:

Given, $\frac{dr}{r} = 0.05 \Rightarrow dr = 0.05r$

Area of circle

$$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$$
$$dA = 2\pi r dr$$
$$\therefore \quad \frac{dA}{A} = \frac{2\pi r dr}{\pi r^2} = 2\frac{dr}{r} = 2 \cdot (0.05) = 0.1$$
$$\therefore \text{ Error in calculating area } = 0.1\%$$

Question102

The stationary points of the curve $y = 8x^2 - x^4 - 4$ are

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Options:

A. $(0, -4), (2, 12), (-2, 12)$

B. $(0, 4), (-2, 12), (1, 2)$

C. $(0, -4), (-1, 2), (2, 12)$

D. $(0, 4), (-1, 2), (1, 2)$

Answer: A

Solution:

$y = 8x^2 - x^4 - 4$

Differentiating w.r.t. x

$\frac{dy}{dx} = 16x - 4x^3 = 4x\left(4 - x^2\right) = 4x(2 - x)(2 + x)$

For stationary points $\frac{dy}{dx} = 0$

$$\begin{array}{lrl} \text{or} & 4x(2-x)(2+x) & = 0 \\ \Rightarrow & x = 0, x = 2, x = -2 & \end{array}$$

At $x = 0, y = -4$

At $x = 2, y = 12$

At $x = -2, y = 12$

Stationary points are $(0, -4), (2, 12), (-2, 12)$.

Question103

The distance between the origin and the normal to the curve $y = e^{2x} + x^2$ drawn at $x = 0$ is units

Options:

A. 2

B. $\frac{2}{\sqrt{3}}$

C. $\frac{2}{\sqrt{5}}$

D. $\frac{1}{2}$

Answer: C

Solution:

Given curve,

$y = e^{2x} + x^2 \Rightarrow dy/dx = 2e^{2x} + 2x$

At $x = 0, y = e^0 + 0 = 1$

and $\frac{dy}{dx} = 2 \cdot e^0 + 0 = 2$

Equation of normal at $(0, 1)$ is

$y - 1 = \frac{-1}{\left(\frac{dy}{dx}\right)_{x=0}}(x - 0)$

$$\begin{aligned} y - 1 &= \frac{-1}{2} \cdot x \\ \Rightarrow \quad 2y - 2 &= -x \\ \Rightarrow \quad x + 2y - 2 &= 0 \end{aligned}$$

Distance between normal to the curve and origin is

$\frac{|1 \cdot 0 + 2 \cdot 0 - 2|}{\sqrt{1^2 + 2^2}} = \frac{2}{\sqrt{5}}$
